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Abstract

In its "Inflation-Overshooting Commitment," the Bank of Japan commits to continuing to expand the monetary base until the year-on-year rate of increase in the CPI exceeds the price stability target of 2 percent and stays above the target in a stable manner. Through the commitment, the Bank of Japan is implementing a so-called "makeup strategy," which aims to offset a part of past inflation misses from the target by allowing actual inflation to overshoot the target for some time and thereby stabilizing average inflation over the business cycle. Existing studies have shown that such makeup strategies are actually effective for the U.S. economy. This paper examines the effectiveness of the makeup strategy for Japan's economy, where inflation expectations formation is known to be largely adaptive. Specifically, we build a small-scale macroeconomic model for Japan's economy and conduct simulation analysis to study the implications of adopting the makeup strategy for early achievement of the inflation target as well as the incurring social welfare costs. Simulation results show that when the inflation rate has been below the target, it is effective to stabilize average inflation by offsetting the past inflation misses over some makeup windows. In addition, the results suggest that when the natural rate of interest is lower, the optimal makeup window becomes longer.

JEL classification: C53, E31, E47, E52, E58.

Key words: Monetary policy, Inflation-overshooting commitment, Makeup strategy, Average inflation targeting, Stochastic simulation.

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1. Introduction

In September 2016, the Bank of Japan (BoJ) introduced "Quantitative and Qualitative Monetary Easing with Yield Curve Control," given its "Comprehensive Assessment" (Bank of Japan, 2016). This monetary policy framework consists of two major components: "Yield Curve Control" and the "Inflation-Overshooting Commitment." The former is a guideline for market operations as the BoJ controls both short-term and long-term interest rates to facilitate the formation of the yield curve deemed most appropriate for achieving the price stability target of 2 percent. In the latter, the BoJ commits to continuing to expand the monetary base until the year-on-year rate of increase in the CPI exceeds the price stability target of 2 percent and stays above the target in a stable manner. The BoJ aims to achieve the price stability target not with a condition where the inflation rate reaches 2 percent only temporarily, but where the inflation rate is 2 percent *on average* over the business cycle. In this sense, the BoJ is implementing a so-called "makeup strategy," which aims to stabilize average inflation by offsetting a part of past inflation misses from the target over some horizons with inflation overshooting.

The Federal Reserve Board (FRB) introduced a flexible form of average inflation targeting (AIT), regarded as one such makeup strategy, in August 2020 (Federal Reserve Board, 2020). In this new monetary policy framework, following periods when inflation has been running below 2 percent, the appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time. Kiley and Roberts (2017) and Bernanke et al. (2019) spell out the benefits of such a makeup strategy: it increases the effects of accommodative monetary policy on the economy and prices in the near future by strengthening market expectations that monetary easing will be prolonged. On the other hand, the authors argue that the makeup strategy also carries a risk of undesirable inflation overshooting and overheating of the economy due to longer periods of monetary easing. Thus both costs and benefits should be evaluated when assessing the overall effectiveness of the makeup strategy.

When the FRB introduced the new monetary policy framework in August 2020, 12 supplementary papers were released. Among them, Arias et al. (2020) and Hebden et al. (2020) examine the effectiveness of the AIT rule using the FRB/US, a large-scale macroeconomic model developed by the FRB. Arias et al. (2020) specify the model equations in the FRB/US, with wage and price setters and financial market participants following model-consistent expectations, and long-run inflation expectations anchored at the Federal Reserve's 2 percent goal. In this setting, they employ a mild demand-drive recession scenario, where the inflation rate drops below the 2 percent goal and an effective lower bound (ELB) is binding, to run simulations under various monetary policy rules. Their results show that the AIT rules, which set the policy interest rate to respond to average past inflation rates, achieve the 2 percent long-run objective earlier than a standard Taylor rule, which responds to the current inflation rate. Furthermore, it is shown that an AIT rule that refers to a longer makeup window, which means a monetary policy rule that makes up past inflation misses over a longer period, achieves the 2 percent goal earlier. These simulation results are consistent with a theoretical prediction that the makeup strategy strengthens the formation of economic agents' forward-looking expectations that monetary easing will be prolonged and thereby increases the stimulative effects of monetary accommodation on the economy and prices (Svensson, 1999; Vestin, 2006).

Hebden et al. (2020) extend the framework of Arias et al. (2020) to the more realistic case, where some economic agents do not follow rational expectations, to examine whether the makeup strategy retains its effectiveness even under this condition. In the FRB/US, they set up the model so that wage and price setters form expectations adaptively and long-run inflation expectations are not well anchored. The analysis assesses whether the AIT rule is still effective in achieving the 2 percent objective earlier. The simulation results indicate that the AIT rule achieves the long-run inflation objective earlier than the Taylor rule, while overshooting 2 percent for some considerable time afterwards. It should be noted, however, that Hebden et al. (2020) do not quantitatively assess the tradeoff in terms of the relative costs and benefits of adopting the makeup strategy and they do not show the optimal length of the makeup window in the AIT rule.

This paper examines whether the makeup strategy is effective in achieving the inflation target of 2 percent earlier for Japan's economy, where the inflation expectations formation is known to be largely adaptive. We conduct simulation analyses with a macroeconomic model which captures the characteristics of Japan's economy. To implement a computationally heavy stochastic simulation with various random shocks, we develop a small-scale macroeconomic model, "S-JEM" (Small-scale Japanese Economic Model), which concisely summarizes the dynamics of major variables in "Q-JEM" (Quarterly Japanese Economic Model), a large-scale macroeconomic model maintained by the Research and Statistics Department at the BoJ.[1] In Q-JEM, a Phillips curve reflects persistently adaptive

[1] As Q-JEM is an econometric model which estimates the historical relationships among Japan's major macroeconomic variables, it is widely utilized for policy evaluation and various risk simulations at the BoJ (see Fukunaga et al., 2011; Hirakata et al., 2019; Kawamoto et al., 2021).

expectations formation, which is one of the major characteristics of Japan's inflation dynamics. The same type of Phillips curve is also adopted in S-JEM. We run simulations starting from an initial state where the ELB is binding, and assess the performance of the economy and prices as well as the social welfare loss under the makeup strategy. It should be noted that in the model simulations the short-term interest rate rule is used as a policy instrument for simplicity when specifying the makeup strategy.

There are three key features that distinguish this paper. First, we conduct not only deterministic simulations that exclude additional shocks but also stochastic simulations that include realistic random shocks. We estimate the distributions of demand and supply shocks based on historical time series data on major economic variables for Japan's economy. We draw random shocks from these distributions and add them to S-JEM to compute the trajectories of the variables. We iterate these stochastic simulations to investigate the average performance of the economy and prices under each monetary policy rule.

Second, we define social welfare loss function in terms of the variances of the economy and inflation and then explore which monetary policy rule minimizes such social welfare costs in Japan's economy. The AIT rule, which makes up, at least in part, the past inflation misses from the inflation target, has the benefit of achieving the inflation target earlier by keeping interest rates low for longer. However, it also carries an accompanying cost: an increased risk that the economy becomes overheated, which could cause undesirable and more prolonged inflation overshooting. Explicitly defining and computing the social welfare loss in the model enables us to quantitatively examine the tradeoff of such costs and benefits under the AIT rule. In addition, we examine two cases with different presumption on the natural rate of interest (r^*), because there is a large uncertainty over the estimates of the natural rate of interest. Specifically, we set the level at (i) $r^* = +0.5\%,$ which is approximately the average of the estimates during the period since 2000; and a lower case, (ii) $r^* = -0.1\%$. We show that the optimal length of the makeup window for the AIT rule crucially depends on the level of the natural rate of interest.

Third, in the stochastic simulations with various realistic shocks, we explore not only the effectiveness of the makeup strategy in terms of the average social welfare loss but also the accompanying tail risk of considerably soaring inflation. We briefly investigate the distributions of the peak inflation rates in each iteration of the simulation to investigate how the makeup strategy increases the tail probability of substantial inflation overshooting.

The remainder of the paper is organized as follows. Section 2 explains the small-scale macroeconomic model, S-JEM. Section 3 documents the simulation methods for our analysis.

Section 4 provides the simulation results. Section 5 concludes. The appendix provides an overview of the existing literature on makeup strategies.

2. Small-scale Japanese macroeconomic model (S-JEM)

2-1. Model setup

We first explain the specifications of the output gap, consumer prices, and nominal long-term interest rates. Note that time frequency in S-JEM is quarterly.

(1) IS Curve

Let y_t denote the output gap. This is specified to be determined by a lagged output gap and a real interest rate gap as follows:

$$y_t \;=\; \alpha_0 y_{t-1} + \alpha_1(i_t^L - \pi_t^{eL} - r^*) + e_t^y, \qquad \alpha_1 < 0, \tag{1}$$

where e_t^y is a demand shock, i_t^L is nominal long-term interest rates (10-year), π_t^{eL} is long-term inflation expectations (10 year average), and r^* is the natural rate of interest. The second term on the right-hand side of equation (1) represents the real long-term interest rate gap. Conventionally, specification of the IS curve includes expected income. However, in Q-JEM, the specification of demand components such as private consumption does not include expected income from the perspective of goodness of fit to time-series data in Japan. Therefore, S-JEM employs the above backward-looking specification following Q-JEM. This formulation implies that we consider monetary policy transmission only through nominal long-term interest rates and long-term inflation expectations.

(2) Philips Curve

In Q-JEM, consumer prices are modeled using a hybrid Phillips curve. In its specification, the consumer price is a function of the output gap and inflation expectations. The latter includes two elements: (i) forward-looking expectations that are formed depending on the pace at which inflation approaches the price stability target of 2 percent; and (ii) backward-looking (i.e., adaptive) expectations that are formed based on realized values of consumer prices. An increase in forward-looking inflation expectations, led by a strengthened commitment to achieving the price stability target, directly contributes to raising consumer prices. If an improvement in the output gap pushes up realized inflation rates and consequently adaptive inflation expectations, inflation rates are further levered up.

Let π_t denote the annualized rate of change in the CPI (all items less fresh food and energy), or the core inflation rate. Following Q-JEM, the core inflation rate is given by

$$\pi_t \;=\; \psi\pi_{t-1} + (1-\psi)\pi_t^{eML} + \kappa y_t, \quad 0 < \psi < 1, \quad \kappa > 0, \tag{2}$$

where π_t^{eML} is medium- to long-term inflation expectations (6 to 10 years ahead). On the right-hand side of equation (2), the first term captures backward-looking expectations in the form of lagged realized core inflation; the second term represents forward-looking expectations formation; and the third term measures the sensitivity of inflation to the output gap with the coefficient κ representing the slope of the Phillips curve.

Let $\tilde{\pi}_t$ denote the annualized rate of change in the CPI (all items), or headline inflation rate, and e_t^{π} denote a price shock such as an energy price shock. We define headline inflation rate as the sum of the core inflation rate and the price shock such that

$$\tilde{\pi}_t \;=\; \pi_t \;+\; e_t^{\pi}. \tag{3}$$

(3) Nominal interest rates

We define the nominal short-term interest rate (the policy rate) by i_t. In S-JEM, we assume nominal short-term interest rates to be determined by a monetary policy rule set by the central bank, and nominal long-term interest rates (10-year) i_t^L in the IS curve (equation (1)) to be determined by the term structure of short-term interest rates. Let $i_{t+h|t}$ denote the expected short-term interest rates at time t for the h-quarter horizon. We then define the nominal long-term interest rates as the averages of the expected short-term interest rates over 10 years (40 quarters) ahead as

$$i_t^L \;=\; \frac{1}{40}\sum_{h=0}^{39} i_{t+h|t}. \tag{4}$$

In this paper, for simplicity, we assume that the term premium for the nominal long-term interest rates is zero.

2-2. Monetary policy rule

Let i_{ELB} denote the effective lower bound (ELB) on the nominal short-term interest rate. Following Q-JEM, we assume that the nominal short-term interest rate is determined by the weighted averages of its lagged value and the policy rate (i_t^s) implied by the monetary policy rule. The nominal short-term interest rate is specified by

$$i_t \;=\; \rho i_{t-1} + (1-\rho)\max[i_t^s,\; i_{\text{ELB}}], \qquad 0 < \rho < 1, \tag{5}$$

where ρ measures the degree of interest rate smoothing (see, e.g., Bernanke et al., 2019). On the right-hand side of equation (5), the max function of the second term takes the value i_{ELB}, if $i_t^s < i_{\text{ELB}}$.

In this paper, we employ the Taylor rule as the baseline monetary policy rule and compare the performance of the AIT rule with that of the Taylor rule. The Taylor rule sets i_t^s so that it responds to the current inflation rate, while under the AIT rule the policy rate responds to the average past inflation rate. We define the average inflation rate over the past n-quarters by

$$\bar{\pi}_t^{(n)} \;=\; \tfrac{1}{n}\sum_{i=0}^{n-1}\pi_{t-i}, \tag{6}$$

where π_t is the annualized core inflation rate defined above. Following this notation, we denote the year-on-year rate of change in the CPI (all items less fresh food and energy, i.e., core inflation rate) by $\bar{\pi}_t^{(4)}$.

(1) Taylor rule

Following Taylor (1993, 1999), we assume that the policy rate is determined by the output gap and the deviation of the inflation rate from its target:

$$i_t^s \;=\; r^* + \bar{\pi}_t^{(4)} + \theta_y y_t + \theta_\pi(\pi_t - \pi^*), \tag{7}$$

where π^* is the inflation target. Note that the sum of the first and the second terms on the right-hand side of equation (7) represents an equilibrium nominal interest rate. The coefficients θ_y and θ_π are the weights placed, respectively, on the output gap and inflation rate in setting the policy rate.

(2) Average inflation targeting

A standard AIT rule used in previous studies (e.g., Arias et al., 2020) is specified as

$$i_t^s \;=\; r^* + \bar{\pi}_t^{(4)} + \theta_y y_t + \theta_\pi N\left(\bar{\pi}_t^{(4N)} - \pi^*\right), \qquad N \geq 1, \tag{8}$$

where $\bar{\pi}_t^{(4N)}$ is the average core inflation rate over the past N years ($4N$ quarters). This equation differs from equation (7) in that the fourth term on the right-hand side includes the N-year average inflation rate ($\bar{\pi}_t^{(4N)}$) instead of the current annualized rate of change in the CPI (π_t). Also, in equation (8), the weight on the inflation rate is $\theta_\pi N$ instead of θ_π. Given

θ_y and θ_π, the AIT puts more weight on the inflation rate than the Taylor rule. Furthermore, as the length of the window (N) increases, more weight is placed on the inflation rate.

Arias et al. (2020) point out that the standard AIT rule given in equation (8) is not always consistent with actual monetary policy: when inflation rates suddenly drop from above to below target, the AIT rule dictates a counterfactually slow cutting of policy rates because it seeks to account for past inflation. To address this issue, Arias et al. (2020) employ an "Asymmetric AIT", which sets the policy rate following a state-dependent rule, depending on whether the average inflation rate is above or below the target. The current paper employs this Asymmetric AIT rule: the standard AIT rule defined by equation (8) is followed when the average inflation rate is below the inflation target; otherwise the Taylor rule defined by equation (7) is followed. That is,

$$i_t^s = \begin{cases} r^* + \bar{\pi}_t^{(4)} + \theta_y y_t + \theta_\pi N\left(\bar{\pi}_t^{(4N)} - \pi^*\right), & \text{if } \bar{\pi}_t^{(4N)} \leq \pi^*, \\ r^* + \bar{\pi}_t^{(4)} + \theta_y y_t + \theta_\pi (\pi_t - \pi^*), & \text{if } \bar{\pi}_t^{(4N)} > \pi^*. \end{cases} \tag{9}$$

2-3. Expectations formation mechanisms

(1) Output gap expectations formation

Let $y_{t+h|t}$ denote the expected output gap at time t for the h-quarter horizon. Following Q-JEM, we assume that output gap expectations formation follows a simple second-order autoregressive (AR(2)) model, given by

$$y_{t+h|t} = \beta_1 y_{t+h-1|t} + \beta_2 y_{t+h-2|t}, \tag{10}$$

where $h = 0, \ldots, 40$, $y_{t-1|t} = y_{t-1}$, and $y_{t-2|t} = y_{t-2}$. A key aspect of this formulation is that the expected output gap is determined only by the realized output gap regardless of the future course of monetary policy. That is, given the realized output gap, the monetary policy rule does not affect the expected output gap. In this sense, the specification reflects the fact that expectations formation is largely adaptive.

(2) Inflation expectations formation

Inflation expectations formation is specified for short-term, medium-term, and long-term expectations. Let $\pi_{t+h|t}$ denote inflation expectations (annual rate) at time t for the h-quarter horizon. We assume the 10-years (40-quarters) ahead expected inflation rate to be determined by the weighted average of the inflation target and the trend inflation rate:

$$\pi_{t+40|t} \;=\; \delta_t \pi^* + (1-\delta_t)\bar{\pi}_t^{(8)}, \tag{11}$$

where $0 < \delta_t < 1$, and $\bar{\pi}_t^{(8)}$ is the average core inflation rate over the past 2 years, regarded as the trend inflation rate in S-JEM. The parameter δ_t measures the degree to which inflation expectations are anchored to the inflation target. The closer to one this parameter is, the more strongly inflation expectations are anchored and the less they are affected by the inflation rate trend. With the anchoring of inflation expectations taking this time-dependent form, we can describe both the situation where inflation expectations stay around the current inflation trend and the situation where they converge to the inflation target. The dynamics of δ_t are specified as follows:

$$\delta_t \;=\; (1-\lambda)\delta_{t-1} + \lambda\left[\max\left\{\underline{\delta},\ \min\left(\overline{\delta}, 1 - a\left|1 - \frac{\pi_{t+39|t-1}}{\pi^*}\right|\right)\right\}\right], \tag{12}$$

where $\overline{\delta}$ and $\underline{\delta}$ are the upper and lower limits of δ_t, i.e., $0 < \underline{\delta} < \overline{\delta} < 1$. The second term on the right-hand side of equation (12) refers to the deviation of the 10-years ahead expected inflation rate in the previous period ($\pi_{t+39|t-1}$) from the inflation target. Parameter a represents the sensitivity of δ_t to that deviation of inflation expectations from the inflation target. Parameter λ measures the persistence of long-term inflation expectations, where $0 < \lambda < 1$.

In equation (12), we assume that economic agents revise δ_t upward when they observe long-term expected inflations relatively close to the inflation target in the previous period. As δ_t rises, the long-term expected inflation rate moves closer to the inflation target, so the actual inflation rate increases reflecting an upward shift of the Phillips curve. This raises the inflation rate trend, which in turn impacts the long-term expected inflation rate as in equation (11) and consequently δ_t is revised upward further. Equations (11) and (12) capture this self-realization mechanism of gathering momentum toward the inflation target in response to an initial upward jolt to inflation expectations.

Regarding medium-term inflation expectations, represented by the 3-years ahead expected inflation rate ($\pi_{t+12|t}$), we assume this to be determined by the lagged headline inflation rate ($\tilde{\pi}_{t-1}$), the 10-years ahead inflation expectations ($\pi_{t+40|t}$), and the 3-years ahead expected output gap ($y_{t+12|t}$). Specifically, we arrange the hybrid Phillips curve mentioned above such that

$$\pi_{t+12|t} \;=\; \psi^M \tilde{\pi}_{t-1} + (1-\psi^M)\pi_{t+40|t} + \kappa^M y_{t+12|t}. \tag{13}$$

We assume that inflation expectations from 3-years ahead to 10-years ahead are simply set by a linear-interpolation of those expected inflation rates (defined in equations (13) and (11), respectively).

Regarding short-term inflation expectations, from the current period to 2-years ahead, we employ a similar hybrid Phillips curve:

$$\pi_{t+h|t} \;=\; \psi^S \pi_{t+h-1|t} + (1-\psi^S)\pi_t^{eML} + \kappa^S y_{t+h|t}, \tag{14}$$

where $h = 0, \dots, 8$, $\pi_{t-1|t} = \pi_{t-1}$, and π_t^{eML} denotes the average expected inflation rate from 6-years to 10-years ahead (i.e., medium- to long-term inflation expectations). We compute the inflation expectations from 2-years to 3-years ahead by a linear-interpolation of the corresponding expected inflation rates (defined in equations (14) and (13), respectively).

Finally, we define the long-term inflation expectations used in the IS curve (equation (1)) as the 10-year average of the expected inflation rate: $\pi_t^{eL} = \frac{1}{40}\sum_{h=0}^{39} \pi_{t+h|t}$.

(3) Expectations formation for nominal short-term interest rates

Regarding the formation of expectations for nominal short-term interest rates, we assume that economic agents formulate expectations of the future course of policy rates anticipating that the current monetary policy rule will remain the same as today. The expected nominal short-term interest rate is derived from equation (5) as follows:

$$i_{t+h|t} \;=\; \rho i_{t+h-1|t} + (1-\rho)\max\left[i_{t+h|t}^S,\; i_{\mathrm{ELB}}\right], \tag{15}$$

where $h = 0, \dots, 40$, and $i_{t-1|t} = i_{t-1}$. Note that we assume that the ELB for nominal interest rates remains the same as today. Let $i_{t+h|t}^S$ denote the expected nominal short-term interest rate implied by the specified monetary policy rule. Under the Taylor rule, the expected rate is derived from equation (7) as follows:

$$i_{t+h|t}^S \;=\; r^* + \bar{\pi}_{t+h|t}^{(4)} + \theta_y y_{t+h|t} + \theta_\pi\left(\pi_{t+h|t} - \pi^*\right). \tag{16}$$

Under the (asymmetric) AIT rule specified in equation (9), economic agents will expect the central bank to follow the AIT rule when the average inflation rate is below the inflation target, but to follow the Taylor rule otherwise. That is,

$$i_{t+h|t}^S \;=\; \begin{cases} r^* + \bar{\pi}_{t+h|t}^{(4)} + \theta_y y_{t+h|t} + \theta_\pi N\left(\bar{\pi}_{t+h|t}^{(4N)} - \pi^*\right), & \text{if } \bar{\pi}_{t+h|t}^{(4N)} \le \pi^*, \\ r^* + \bar{\pi}_{t+h|t}^{(4)} + \theta_y y_{t+h|t} + \theta_\pi\left(\pi_{t+h|t} - \pi^*\right), & \text{if } \bar{\pi}_{t+h|t}^{(4N)} > \pi^*. \end{cases} \tag{17}$$

(4) Summary

In summary, the difference between monetary policy rules such as the Taylor rule and the AIT rule affects the future course of expected nominal short-term interest rates and nominal long-term interest rates through the term structure. The difference in nominal long-term interest rates causes different trajectories of the output gap through the IS curve (equation (1)) and of the inflation rate through the Phillips curve (equation (2)). The realized output gap and inflation rates affect the expected output gap and inflation expectations as well as the trajectory of expected nominal short-term interest rates.

3. Simulation Method

In this section, we explain the simulation method for our analysis using S-JEM. The parameters in the monetary policy rules are set as follows: the smoothing parameter on the nominal short-term interest rate is $\rho = 0.9$; the weights on the output gap and inflation rate are $\theta_y = 0.5$, $\theta_\pi = 1.0$, respectively; and the inflation target is 2 percent, in line with the BoJ's price stability target. The ELB on the nominal interest rate is set as $i_{\mathrm{ELB}} = -0.1\%$. The results do not change significantly when this value is set lower as far as there are episodes of the ELB binding in simulated paths.

The initial values of the nominal short-term interest rates, the inflation rate, and the output gap are set as $i_0 = -0.1\%$, $\pi_0 = +0.7\%$, and $y_0 = +0.3\%$. For simplicity, the initial value of the inflation rate reflects the median of the Policy Board members' forecasts of CPI inflation for fiscal 2022 announced in the January 2021 Outlook for Economic Activity and Prices. We set the value of the output gap based on the simple linear regression of the long-term relationship between the output gap and the inflation rate (i.e., Phillips curve) provided in the above Outlook, by inserting the initial value of the inflation rate into the regression. The initial value of long-term inflation expectations is set as $\pi_{40|0} = +1.3\%$, referencing the 6-year to10-year inflation expectations from the Consensus forecast in the second half of 2020, and the initial value of the parameter anchoring long-term inflation expectations to the inflation target as 50%, that is, $\delta_0 = 0.5$. The initial value of the nominal long-term interest rate is set as $i_t^L = 0.0\%$, referencing its actual values in the second half of 2020.

We calibrate the parameters in each equation of the model using time-series data in Japan as well as referencing corresponding parameters in Q-JEM. Parameters are listed in Chart 1. The calibrated model reflects the persistently adaptive formation of inflation expectations because the coefficient on the backward-looking component in the Phillips curve (equation (2)), ψ is as large as 0.85, and the parameter anchoring inflation expectations to the inflation target is specified as being below one. Regarding the natural rate of interest, we refer to estimates in Q-JEM to set two cases in our simulation analysis: (a) $r^* = +0.5\%$, which is approximately the average of the estimates during the period between 2000 and 2020; and a lower case, (b) $r^* = -0.1\%$, the average during the period between 2016 and 2020.

In the following simulation analysis, we firstly compute trajectories of the economic variables in the model under each monetary policy rule without any additional shock. This experiment assesses how each monetary policy rule affects the paths of the economic variables from their above-mentioned initial values. We then conduct a stochastic simulation with random shocks following the distributions of demand and price shocks estimated from the time-series data in Japan. Specifically, we assume demand and price shocks are distributed as follows.

$$e_t^y = \varepsilon_t^y + \varphi_y \varepsilon_{t-1}^y, \quad \varepsilon_t^y \sim N(\mu_y, \sigma_y^2), \tag{18}$$

$$e_t^\pi = \varepsilon_t^\pi + \varphi_\pi \varepsilon_{t-1}^\pi, \quad \varepsilon_t^\pi \sim N(\mu_\pi, \sigma_\pi^2). \tag{19}$$

where the parameters are calibrated as in Chart 1.

In the stochastic simulation with random shocks, the average performances of economic activity and prices in Japan under each monetary policy rule are computed by simulating 10 years (40 quarters) of economic activity from the above-mentioned initial values over 1,000 iterations. We define a simple social welfare loss function to evaluate the effectiveness of each monetary policy rule from the perspective of assessing both costs and benefits of the makeup strategy. In line with the existing literature, the loss function is defined as the squared sum of the fluctuation of the output gap and the deviation of the inflation rate from the inflation target:

$$L = \frac{1}{KT}\sum_{i=1}^{K}\sum_{t=1}^{T}\left\{\left(y_t^{[i]}\right)^2 + \left(\pi_t^{[i]} - \pi^*\right)^2\right\}, \tag{20}$$

where $y_t^{[i]}$ and $\pi_t^{[i]}$ are the output gap and the inflation rate, respectively, at time t in the i-th simulation; T denotes the length of simulation period (10 years) in each iteration; and K denotes the number of simulation iterations (1,000 times). We simply set equal weights on the loss function (equation (20)) because there is no consensus in the existing literature about the relative weights to be placed on the fluctuation of the output gap and the deviation of the inflation rate from the inflation target.

4. Simulation Results

4-1. Baseline result

Charts 2 and 3 plot results of the simulation without any shock for the cases where the natural rate of interest, $r^* = +0.5\%$ and -0.1%, respectively. The figures show how each of the main economic variables evolves under the Taylor rule and three different specifications of the AIT rule, with two-, three-, and four-year makeup windows, respectively.

Chart 2 shows that nominal short-term interest rates are more accommodative under the AIT rules than the Taylor rule in the first half of the simulation period. As the length of the makeup window increases, nominal short-term interest rates become more accommodative. The nominal long-term interest rates are lower in the AIT rule with the longer window in the early stage of the simulation period, reflecting the term structure of expected short-term interest rates. Due to this difference in nominal long-term interest rates, the output gap increase is larger in the AIT rule with the longer window, which affects inflation rates through the Phillips curve. As a result, the inflation target is seen to be achieved earlier under the AIT rule than the Taylor rule. As the length of the makeup window increases, the inflation target is achieved earlier. However, the AIT rule with the longer window causes increased overheating of the economy and inflation overshooting above the target of 2 percent due to the prolonged monetary easing. Chart 3 shows that when the natural rate of interest $r^* = -0.1\%$, the increases in the output gap and inflation rate are less marked than when $r^* = +0.5\%$, because the effects of monetary policy are smaller. Still, the qualitative differences in the performance of the economy and prices between the Taylor rule and the AIT rules remains unchanged from those observed in Chart 2.

Chart 4 reports the social welfare losses under each monetary policy rule computed in stochastic simulations with random shocks. The AIT rule with a 2-year makeup window is seen to minimize the social welfare loss when the natural rate of interest $r^* = +0.5\%$, and the AIT with a 3-year window when $r^* = -0.1\%$. This result suggests that, in Japan, when observed inflation is below the inflation target, monetary policy should pay heed to past inflation misses not only from the perspective of achieving the inflation target earlier but also with a view to minimizing social welfare costs. Furthermore, when the natural rate of interest is low, the results suggest including a longer period of past inflation in the policy rule, increasing the "makeup" effect to compensate for the weaker effects of monetary easing in such an environment.

4-2. Robustness check

We examine the robustness of the baseline results, conducting stochastic simulations for the following three cases.

First, we run a stochastic simulation with the interest rate sensitivity of the output gap ($-\alpha_1$) in the IS curve (equation (1)) reduced from 0.4 to 0.3. This interest rate sensitivity parameter crucially affects the performance of each monetary policy rule through the differing trajectories of the output gap and the inflation rates as long-term interest rates follow pathways determined by the term structure of short-term interest rates in S-JEM. Chart 5 shows the social welfare loss under the AIT rules to be less than that under the Taylor rule for both cases of the natural rate of interest. The AIT with a 3-year window minimizes the social welfare loss when the natural rate of interest $r^* = +0.5\%$, and the AIT with a 5-year window proves optimal when $r^* = -0.1\%$. Compared with Chart 4, the results in Chart 5 suggest that monetary policy effectiveness diminishes when the output gap is less sensitive to changes in interest rates, so that the makeup window that minimizes the social welfare loss is longer than in the baseline result.

Second, we analyze the case where the Phillips curve, which measures the relationship between inflation and economic slack, is steeper than in the baseline case. Specifically, we change the values of $(\kappa, \kappa^M, \kappa^S)$ from 0.07 to 0.12. The slope of the Phillips curve is also a key parameter because it impacts the effect of the monetary policy on the inflation rate in S-JEM. Chart 6 shows the social welfare loss under the AIT rules to be less than that under the Taylor rule for both cases of the natural rate of interest. The AIT with a 3-year window minimizes the social welfare loss when the natural rate of interest $r^* = +0.5\%$, while the AIT with a 5-year window minimizes the social welfare loss when $r^* = -0.1\%$. As the Phillips curve steepens, on the one hand (a) inflation declines more in response to the same size of negative demand shock and the economy achieves the inflation target of 2 percent later; but on the other hand, (b) inflation pressure from an equivalent increase in the output gap as a result of monetary easing is larger. Comparing the baseline result in Chart 4, Chart 6 indicates that the former channel tends to outweigh the latter channel, making the AIT rule with the longer window more effective from the perspective of minimizing social welfare costs.

Third, we conduct a stochastic simulation starting from randomly selected initial values and compare the results with the baseline case where the initial values were fixed. We randomly draw initial values for inflation, the output gap, the 10-years ahead inflation expectations, and the degree to which inflation expectations are anchored to the inflation target from uniform distributions where the range of deviation from the baseline initial values is plus/minus 0.3% points. This exercise assesses the sensitivity of the model simulation to the initial values. Chart 7 reports social welfare losses, which are seen to be lower under the AIT rules than under the Taylor rule for both cases of the natural rate of interest. The AIT rule with the 2-year window minimizes the social welfare loss when the natural rate of interest $r^* = +0.5\%$, and the AIT rule with the 3-year window does so when $r^* = -0.1\%$, which is the same result as in the baseline.

The results of these robustness checks confirm that while the length of the makeup window that minimizes the loss function changes depending on the parameters of the model, the AIT rules, with their commitment to a makeup strategy, remain preferable to the Taylor rule in terms of social welfare costs. Furthermore, for lower natural rates of interest, the optimal makeup window that minimizes the loss function becomes longer.

4-3. Tail risk assessment

We assess the risk of undesirable inflation overshooting when an AIT rule prolongs the period of low interest rates, even though this is expected to happen only with low probability. We investigate the inflation rate peaks under the AIT rule that minimizes the average social welfare loss in the baseline setting. We then evaluate how these inflation rate peaks change under an AIT rule with a longer makeup window.

Chart 8 plots the distributions of the peak inflation rates in each 10-year iteration over 1,000 iterations of stochastic simulation. In the case of $r^* = +0.5\%$, the peak inflation rate is mostly between 2 and 3 percent under the AIT rule with a 2-year window. However, this rule carries a risk of inflation reaching 3 or 4 percent depending on the shocks, though the probability of such peak inflation being realized is low. Under an AIT rule with a longer window, such as 6 years, the distribution of peak inflation rates not only shifts higher but also has a heavier right tail. In the case of $r^* = -0.1\%$, where monetary policy is correspondingly less effective, the probability of the tail risk materializing is lower. The distribution under the AIT rule with a 6-year window has a heavier right tail than the AIT rule with a 3-year window. The difference between two distributions appears to be smaller than that in the case of $r^* = +0.5\%$.

5. Concluding remarks

Extending the existing literature on makeup strategies, which focus on the U.S. context, this paper build a small-scale macroeconomic model to examine whether the makeup strategy is effective in achieving the inflation target of 2 percent earlier for Japan's economy, where the mechanism of inflation expectations formation is known to be largely adaptive. Simulation results show that when the inflation rate has been below the inflation target, it is effective to conduct monetary policy so as to "make up" a part of past inflation misses from the target. Such a makeup strategy is also effective from the perspective of minimizing social welfare costs. In addition, the results suggest that when the natural rate of interest is lower, the optimal makeup window that minimizes the social welfare cost becomes longer.

A caveat in this paper is that, for the sake of model tractability, we use quite a simple macroeconomic model and include only the short-term interest rate as monetary policy tool, which is not necessarily a precise reflection of actual economic mechanisms or how monetary policy is conducted. In particular, we are fully cognizant of how sensitive the length of the AIT makeup window that minimizes social welfare losses is to the model's parameter values and the specification of the social welfare loss function.

Appendix. Literature on makeup strategies

This appendix summarizes major existing studies on makeup strategies. The studies have been developed in the context of a discussion of policy options when there is no room to cut policy interest rates further (the so-called liquidity trap), suggesting that in such cases it is important for central banks to work on the expectations of economic agents and to commit to sustaining monetary easing in the future.

Reifschneider and Williams (2000) provide simulation analyses using the FRB/US and show that makeup strategies that incorporate past inflation misses from target perform better than the Taylor rule in a low interest rate environment. Eggertsson and Woodford (2003) develop a theoretical model and argue that history-dependent policy rules maximize social welfare in a liquidity trap, rather than policy rules such as the Taylor rule that respond to shocks with immediate changes in the policy interest rate.

One of major makeup strategies is price level targeting (PLT). The Taylor rule determines policy interest rates in response to the deviation between the actual inflation rate and the inflation target. In contrast, policy interest rates in a PLT rule are determined by the deviation between the actual price level and what the ideal price level would be assuming the inflation target had been continuously met over a given period. When the price level is below the price level target, a central bank promises to conduct accommodative monetary policy to make up the accumulated misses in the price level; and this pushes inflation rates above the inflation target for a compensatory period. Svensson (1999) and Vestin (2006) develop theoretical models to show that the PLT rule is more effective than the Taylor rule in terms of social welfare (see also, Evans, 2012; Walsh, 2019; and Svensson, 2020)

Nessén and Vestin (2005) specify the AIT rule and analyze it using a theoretical model. Bernanke (2017) points out that, in terms of its underlying thinking, AIT can be classified as a kind of flexible PLT, which he calls "Temporary PLT" (see also, Clarida, 2020). Hebden and López-Salido (2018) examine the effectiveness of the AIT rule and similar makeup strategies, using a small-scale macroeconomic model based on the FRB/US (see also, Bernanke, 2020). Mertens and Williams (2019) also study the effectiveness of makeup strategies such as PLT and AIT, using a theoretical model.

References

Arias, Jonas, Martin Bodenstein, Hess Chung, Thorsten Drautzburg, and Andrea Raffo (2020), "Alternative strategies: How do they work? How might they help?" Finance and Economics Discussion Series 2020-068, Board of Governors of the Federal Reserve System.

Bank of Japan (2016), "New framework for strengthening monetary easing: 'Quantitative and Qualitative Monetary Easing with Yield Curve Control'," September 21, 2016.

Bernanke, Ben (2017), "Monetary policy in a new era," presented at a conference on "Rethinking Macroeconomic Policy," Peterson Institute for International Economics, Washington, October 12–13.

Bernanke, Ben (2020), "The new tools of monetary policy," *American Economic Review*, 110(4), pp. 943–983.

Bernanke, Ben, Michael Kiley, and John Roberts (2019), "Monetary policy strategies for a low-rate environment," Finance and Economics Discussion Series 2019-009, Board of Governors of the Federal Reserve System.

Clarida, Richard (2020), "The Federal Reserve's new framework: Context and consequences," speech at "The Economy and Monetary Policy," Hutchins Center on Fiscal and Monetary Policy at the Brookings Institution, Washington, November 16.

Eggertsson, Gauti, and Michael Woodford (2003), "The zero bound on interest rates and optimal monetary policy," *Brookings Papers on Economic Activity*, 2003(1), pp. 139–211.

Evans, Charles (2012), "Monetary policy in a low-inflation environment: Developing a state-contingent price-level target," *Journal of Money, Credit and Banking*, 44 (S1), pp. 147–55.

Federal Reserve Board (2020), "Federal Open Market Committee announces approval of updates to its Statement on Longer-Run Goals and Monetary Policy Strategy," August 27, 2020.

Fukunaga, Ichiro, Naoko Hara, Satoko Kojima, Yoichi Ueno, and Shunichi Yoneyama (2011), "The Quarterly Japanese Economic Model (Q-JEM): 2011 version," Bank of Japan Working Paper Series, No. 11-E-11.

Hebden, James, Edward Herbst, Jenny Tang, Giorgio Topa, and Fabian Winkler (2020), "How robust are makeup strategies to key alternative assumptions?" Finance and Economics Discussion Series 2020-069, Board of Governors of the Federal Reserve System.

Hebden, James, and J. David López-Salido (2018), "From Taylor's rule to Bernanke's temporary price level targeting," Finance and Economics Discussion Series 2018-051, Board of Governors of the Federal Reserve System.

Hirakata, Naohisa, Kazutoshi Kan, Akihiro Kanafuji, Yosuke Kido, Yui Kishaba, Tomonori Murakoshi, and Takeshi Shinohara (2019), "The Quarterly Japanese Economic Model (Q-JEM): 2019 version," Bank of Japan Working Paper Series, No. 19-E-07.

Kawamoto, Takuji, Takashi Nakazawa, Yui Kishaba, Kohei Matsumura, and Jouchi Nakajima (2021), "Estimating effects of expansionary monetary policy since the introduction of Quantitative and Qualitative Monetary Easing (QQE) using the macroeconomic model (Q-JEM)," Bank of Japan Working Paper Series, No. 21-E-4.

Kiley, Michael, and John Roberts (2017), "Monetary policy in a low interest rate world," *Brookings Papers on Economic Activity*, 2017(1), pp. 317–396.

Mertens, Thomas, and John Williams (2019), "Tying down the anchor: Monetary policy rules and the lower bound on interest rates," Staff Reports 887, Federal Reserve Bank of New York.

Nessén, Marianne, and David Vestin (2005), "Average inflation targeting," *Journal of Money, Credit and Banking*, 37(5), pp. 837–863.

Reifschneider, David, and John Williams (2000), "Three lessons for monetary policy in a low-inflation era," *Journal of Money, Credit and Banking*, 32(4), pp. 936–966.

Sudo, Nao, Yosuke Okazaki, and Yasutaka Takizuka (2018), "Determinants of the natural rate of interest in Japan: Approaches based on a DSGE model and OG model," *Bank of Japan Research Laboratory Series*, No. 18-E-1.

Svensson, Lars (1999), "Price-level targeting versus inflation targeting: A free lunch?" *Journal of Money, Credit and Banking*, 31(3), pp. 277–295.

Svensson, Lars (2020), "Monetary policy strategies for the Federal Reserve," *International Journal of Central Banking*, 16, pp. 133–193.

Taylor, John (1993), "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195–214.

Taylor, John (1999), "An historical analysis of monetary policy rules," in John Taylor, ed., *Monetary Policy Rules*, University of Chicago Press.

Vestin, David (2006), "Price-level versus inflation targeting," *Journal of Monetary Economics*, 53(7), pp. 1361–1376.

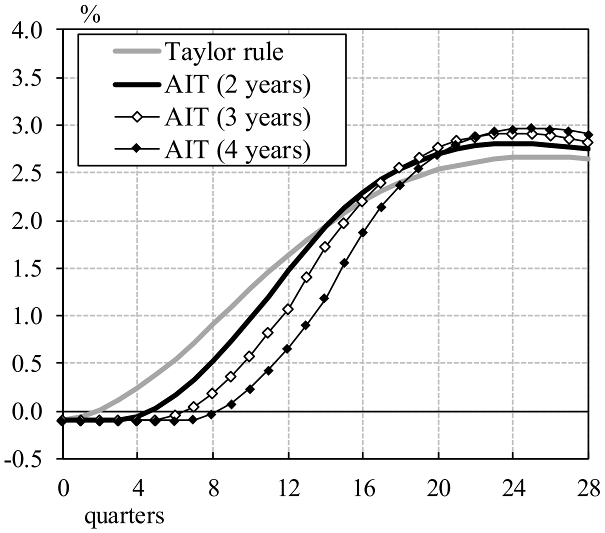
Walsh, Carl (2019), "Alternatives to inflation targeting in low interest rate environments," IMES Discussion Paper Series, No. 19-E-13, Institute for Monetary and Economic Studies, Bank of Japan.

Chart 1. Parameter values

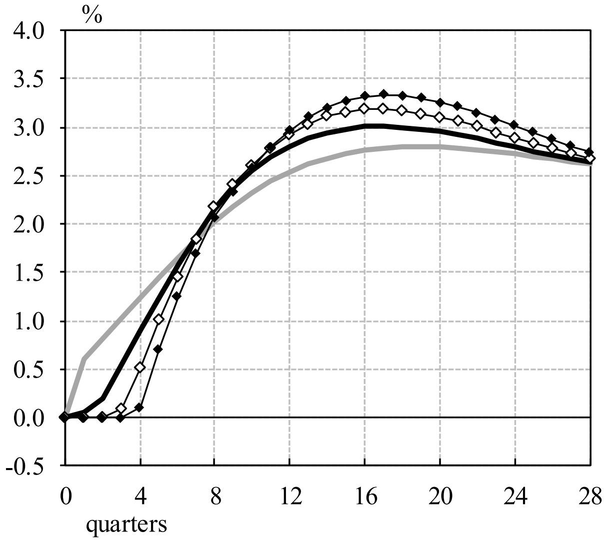
Function	Parameter	Value
IS curve	α_0	0.9
	α_1	−0.4
Phillips curve	ψ	0.85
	κ	0.07
Monetary policy rule	ρ	0.9
	θ_y	0.5
	θ_π	1.0
Output gap expectations formation	β_1	1.44
	β_2	−0.52
10-year ahead inflation expectations	λ	0.3
	a	1.0
	$\overline{\delta}$	0.95
	$\underline{\delta}$	0.05
Medium-term inflation expectations	ψ^M	0.85
	κ^M	0.07
Short-term inflation expectations	ψ^S	0.85
	κ^S	0.07
Distribution of demand shocks	φ_y	0.3
	μ_y	−0.15
	σ_y^2	0.8
Distribution of price shocks	φ_π	0.3
	μ_π	−0.2
	σ_π^2	1.2

Chart 2. Simulation results (natural rate of interest: +0.5%)

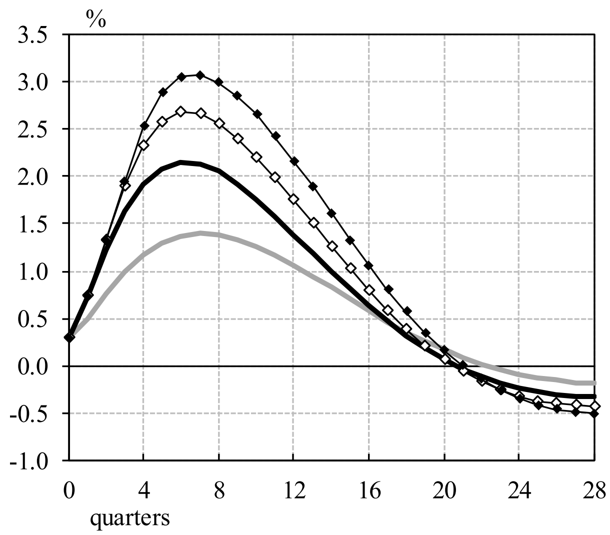
(a) Nominal short-term interest rates



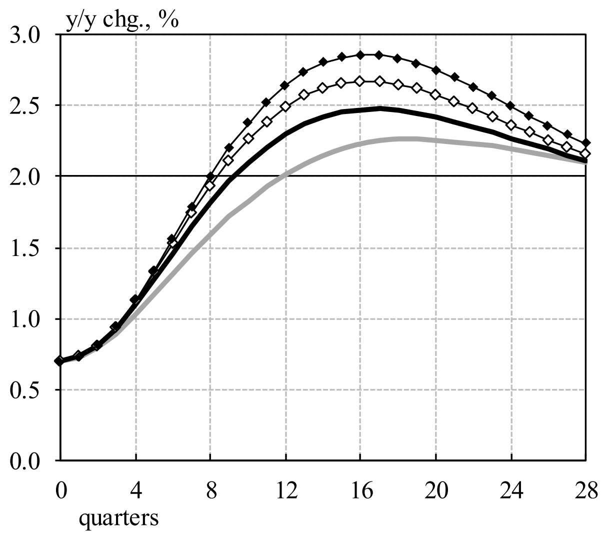
(b) Nominal long-term interest rates



(c) Output gap

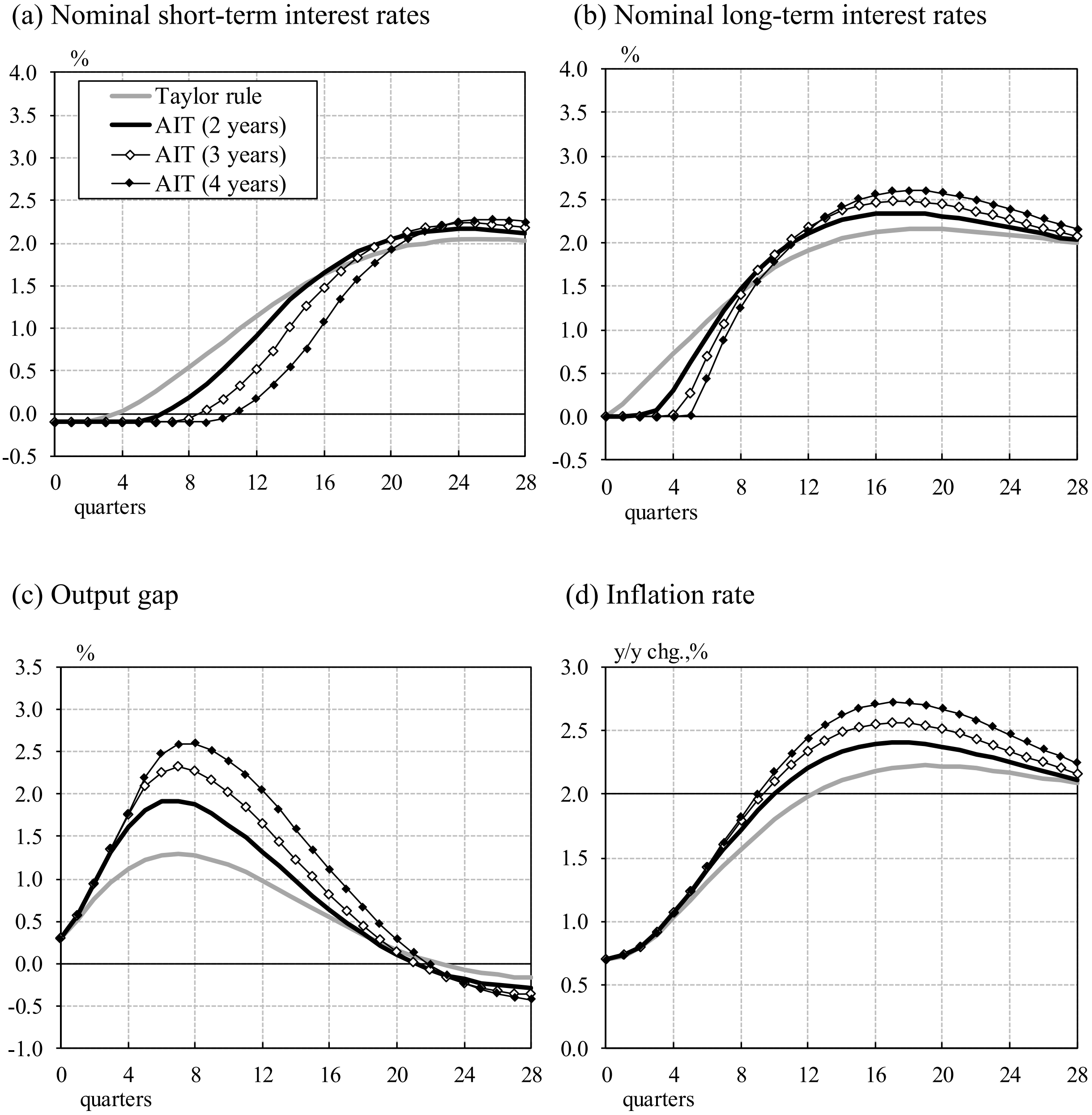


(d) Inflation rate



Note: In the caption, the length of the makeup window for each AIT rule is shown in parentheses. The inflation rate in (d) is $\bar{\pi}_t^{(4)}$ in the model.

Chart 3. Simulation results (natural rate of interest: −0.1%)



Note: In the caption, the length of the makeup window for each AIT rule is shown in parentheses. The inflation rate in (d) is $\bar{\pi}_t^{(4)}$ in the model.

Chart 4. Results of stochastic simulations

(a) Natural rate of interest: +0.5%

Policy rule	Loss	V[y]	V[p]
Taylor rule	5.40	4.43	0.97
AIT (1 year)	5.31	4.38	0.93
AIT (2 years)	4.84	4.20	0.65
AIT (3 years)	5.01	4.42	0.59
AIT (4 years)	5.34	4.74	0.60
AIT (5 years)	5.77	5.13	0.64
AIT (6 years)	6.29	5.58	0.71

(b) Natural rate of interest: −0.1%

Policy rule	Loss	V[y]	V[p]
Taylor rule	7.21	5.68	1.53
AIT (1 year)	7.02	5.56	1.47
AIT (2 years)	6.01	4.91	1.10
AIT (3 years)	5.83	4.84	0.99
AIT (4 years)	5.86	4.92	0.94
AIT (5 years)	6.00	5.07	0.93
AIT (6 years)	6.23	5.30	0.94

Note: The length of the makeup window for each AIT rule is shown in parentheses. Loss is calculated from the loss function averaged over the 1,000 iterations of each stochastic simulation. V[y] and V[p] are the contributions to the loss function of the variances of the output gap and inflation rate, respectively. The policy rule that minimizes the loss within each table is shaded.

Chart 5. Robustness check (1): sensitivity of interest rate ($-\alpha_1$) changed from 0.4 to 0.3

(a) Natural rate of interest: +0.5%

Policy rule	Loss	V[y]	V[p]
Taylor rule	6.47	5.10	1.36
AIT (1 year)	6.36	5.04	1.32
AIT (2 years)	5.57	4.61	0.96
AIT (3 years)	5.50	4.63	0.86
AIT (4 years)	5.62	4.78	0.83
AIT (5 years)	5.83	4.99	0.84
AIT (6 years)	6.13	5.26	0.86

(b) Natural rate of interest: −0.1%

Policy rule	Loss	V[y]	V[p]
Taylor rule	9.03	6.85	2.18
AIT (1 year)	8.86	6.74	2.12
AIT (2 years)	7.81	6.07	1.74
AIT (3 years)	7.53	5.91	1.62
AIT (4 years)	7.43	5.87	1.55
AIT (5 years)	7.42	5.91	1.51
AIT (6 years)	7.48	5.99	1.50

Note: The length of the makeup window for each AIT rule is shown in parentheses. Loss is calculated from the loss function averaged over the 1,000 iterations of each stochastic simulation. V[y] and V[p] are the contributions to the loss function of the variances of the output gap and inflation rate, respectively. The policy rule that minimizes the loss within each table is shaded.

Chart 6. Robustness check (2):
slope of Phillips curve (κ) changed from 0.07 to 0.12

(a) Natural rate of interest: +0.5%

Policy rule	Loss	V[y]	V[p]
Taylor rule	7.41	4.93	2.48
AIT (1 year)	6.86	4.63	2.23
AIT (2 years)	5.10	3.82	1.27
AIT (3 years)	5.03	3.88	1.14
AIT (4 years)	5.19	4.04	1.14
AIT (5 years)	5.47	4.27	1.20
AIT (6 years)	5.86	4.55	1.31

(b) Natural rate of interest: −0.1%

Policy rule	Loss	V[y]	V[p]
Taylor rule	12.12	7.89	4.23
AIT (1 year)	11.17	7.32	3.86
AIT (2 years)	7.11	4.97	2.14
AIT (3 years)	6.38	4.61	1.78
AIT (4 years)	6.14	4.52	1.63
AIT (5 years)	6.12	4.55	1.57
AIT (6 years)	6.24	4.67	1.58

Note: The length of the makeup window for each AIT rule is shown in parentheses. Loss is calculated from the loss function averaged over the 1,000 iterations of each stochastic simulation. V[y] and V[p] are the contributions to the loss function of the variances of the output gap and inflation rate, respectively. The policy rule that minimizes the loss within each table is shaded. The values of κ^M and κ^S are altered in the same way as κ.

Chart 7. Robustness check (3): initial conditions randomly generated

(a) Natural rate of interest: +0.5%

Policy rule	Loss	V[y]	V[p]
Taylor rule	5.35	4.39	0.96
AIT (1 year)	5.26	4.33	0.92
AIT (2 years)	4.82	4.18	0.64
AIT (3 years)	5.00	4.41	0.59
AIT (4 years)	5.34	4.73	0.60
AIT (5 years)	5.77	5.12	0.65
AIT (6 years)	6.29	5.58	0.72

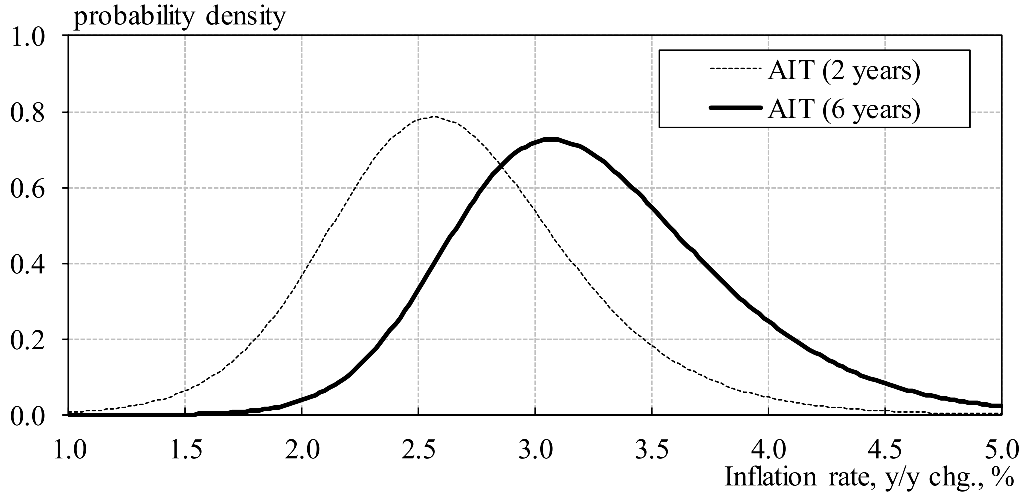
(b) Natural rate of interest: −0.1%

Policy rule	Loss	V[y]	V[p]
Taylor rule	7.37	5.80	1.57
AIT (1 year)	7.19	5.67	1.51
AIT (2 years)	6.15	5.02	1.14
AIT (3 years)	5.97	4.94	1.03
AIT (4 years)	6.00	5.02	0.98
AIT (5 years)	6.12	5.16	0.96
AIT (6 years)	6.35	5.38	0.97

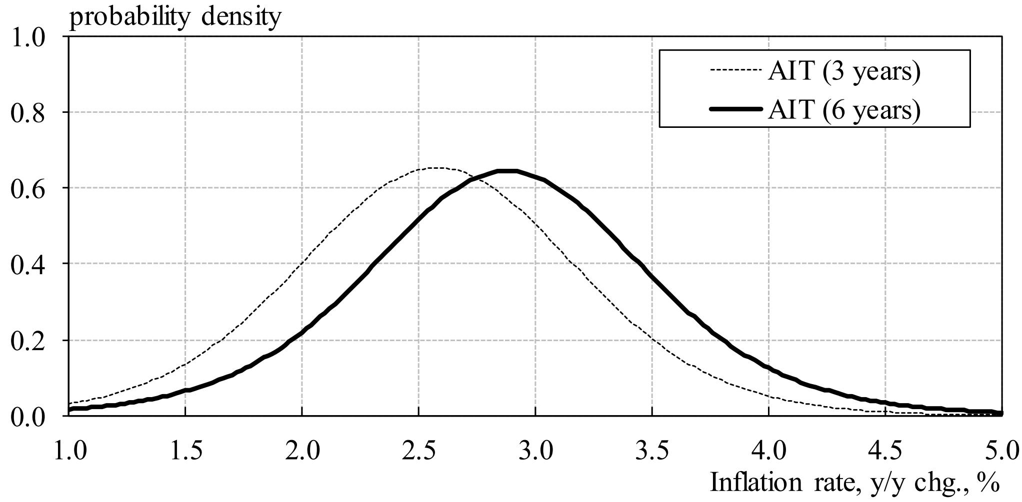
Note: The length of the makeup window for each AIT rule is shown in parentheses. Loss is calculated from the loss function averaged over the 1,000 iterations of each stochastic simulation. V[y] and V[p] are the contributions to the loss function of the variances of the output gap and inflation rate, respectively. The policy rule that minimizes the loss within each table is shaded.

Chart 8. Results of stochastic simulations: distribution of peak inflation rates

(a) Natural rate of interest: +0.5%



(b) Natural rate of interest: −0.1%



Note: Distributions are of the peak inflation rate over the 1,000 iterations of each stochastic simulation. The inflation rate is $\bar{\pi}_t^{(4)}$ in the model.