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BANKS' STOCKHOLDINGS AND THE CORRELATION BETWEEN BONDS AND STOCKS:*

A PORTFOLIO THEORETIC APPROACH

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ABSTRACT

In this paper, we analyze the optimal asset composition ratio of stocks and bonds for a bank taking into consideration the correlation between the interest rate risk and equity risk in the financial capital market using a portfolio model. The analysis reveals that in determining the asset composition ratio in Japan, the correlation coefficient between the interest rate and stock prices as well as the stock price volatility plays a more important role than the interest rate volatility. We also show that in the present circumstances, the stockholding ratios of most financial institutions in Japan are higher than the levels calculated from the model. It is suggested that when the market is exposed to severe stress such as a surge in stock price volatility or reversal of the correlation between the interest rate and stock prices, the stockholding ratios would be even more excessive than the levels obtained from the model.

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1. Introduction

While a bank earns profits by holding a variety of assets including bonds, stocks and loans, it is also exposed to the risk associated with price fluctuations of these assets. It is therefore necessary to pay particular attention to the risk management in stockholding by the Japanese banks. Though the banks reduced their stockholdings through the 2000s, the outstanding amount of stocks held by financial institutions remains high and the equity risk represents a significant risk factor for bank management (Figures 1 and 2).

In Japan since the 2000s, the interest rate risk and the equity risk have been negatively correlated, albeit slightly, so banks are believed to have enjoyed a risk hedging effect by simultaneously holding stocks and bonds (Figure 3). That is, in most cases, when the interest rate rose, unrealized losses on bonds were set off by unrealized gains on stocks. As such risk hedging effect is produced between asset classes, the loss of the overall portfolio can be reduced to a lower level than would be expected from individual risks. On the other hand, in Japan in the 1990s after the bursting of the bubble economy and in Italy around the end of 2011 when concerns grew about the sovereign risk, there were times when stock prices declined significantly while bond interest rates rose at the same time (i.e., simultaneous declines in stock and bond prices) (Figure 4). In such a case, the loss would be greater than expected because the interest rate risk would be positively correlated with the equity risk and both asset classes would produce losses. Such cases indicate that the total risk faced by a bank may be over- or under-estimated if the risk of each asset class is separately evaluated and simply summed up. Therefore, banks face a difficult problem of determining the asset composition so as to maximize profits while striking a balance among the risk of different asset classes under capital constraints.

There have been many studies on the asset composition (portfolio) optimization problem, including the study by Markowitz (1959). Among others, studies which analyzed a mixed portfolio of bonds and stocks include those by Konno and Kobayashi (1997) and Fischer and Roehrl (2003). Konno and Kobayashi (1997) derived the optimal allocation for a large-scale portfolio comprised of bonds and stocks through simulation. Fischer and Roehrl (2003), when they posed a portfolio selection problem, considered a number of optimization criteria including the expected shortfall and RORAC and performed a comparative analysis of their performances through simulation[1].

The purpose of this paper is to obtain the optimal asset composition ratio of bonds and stocks based on the portfolio model for a bank when the bank faces a capital constraint. The correlation between the equity risk and the interest rate risk in Japan is expressly taken into consideration. This paper also analyzes how the asset composition ratio changes in response to stress imposed on the market environment. When the market is in times of stress such as a financial crisis, for example, the hedging effect of stock and bond holdings may be lost due to a sharp decline in stock prices and the reversal of the correlation between the interest rate and stock prices. In such a case, the optimal asset composition ratio also changes.

This paper is organized into five sections. Section 2 describes the model of the optimization behavior of the banks' bond and stockholding ratios. The correlation between bonds and stocks is expressly considered. Section 3 shows the qualitative results obtained from the estimated model. Section 4 compares the data and the results obtained from the model and attempts to assess whether the stockholding ratio of a financial institution is appropriate relative to the capital buffer. Section 5 contains the conclusions and future issues.

[1] RAROC (Risk Adjusted Return On Capital) is an indicator which expresses the earning strength relative to the allocated capital, calculated as “profits after expenses/economic capital”. “Economic capital” here may refer to the amount of risk or to the allowable maximum amount of risk.

2. Analytical Method

2.1 Model

The analysis assumes a portfolio model comprised of two financial instruments: stocks and bonds[2]. Of these, for the bond portfolio, the bank is assumed to trade bonds with duration D. The bank is also assumed to determine the holding ratios of stocks and bonds so as to maximize the expected return of portfolio while controlling the total amount of risk of the portfolio containing the equity risk and interest rate risk at or below the allowable level relative to the capital constraint.

If the present time is 0 and the investment duration is T, the bank's investment strategy can be expressed as[3]:

$$\max_{\{\boldsymbol{w}\}} \mathbf{E}_0 \left[w_1 \left(\frac{B_{T,D} + c_{0,D} T}{B_{0,D}} - 1 \right) + w_2 \left(\frac{S_T}{S_0} - 1 \right) \right], \tag{1}$$

s.t.

$$\mathbf{V}_0[\phi_T] \leq \gamma, \quad \gamma > 0 \tag{2}$$

$$w_1 + w_2 = 1 \tag{3}$$

where, $\mathbf{E}_0[]$ and$\mathbf{V}_0[]$ represent the expectation and the variance at time 0 respectively. $B_{T,D}$ and S_T represent the value of the bond and stock portfolio at time T, respectively. $\phi_T \equiv w_1\big((B_{T,D} + c_{0,D}T)/B_{0,D} - 1\big) + w_2(S_T/S_0 - 1)$ is the rate of return from the portfolio from time 0 to T, $\boldsymbol{w} = (w_1, w_2)$ is the investment weight on bonds/stocks, $c_{0,D}$ is the coupon amount of the bond at maturity $c_{0,D}$[4], and γ is the variance (volatility) of the allowable

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[2] There are three major types of a bank's key asset holdings: loans, bonds and stocks. The analysis in this paper assumes that banks determine the loans first, and on that basis, address the problem of how to allocate the remaining assets to bonds and stocks.

[3] In setting the problem, the constraint condition $w_1, w_2 \geq 0$ is not imposed. That is, the assessment is made on the basis that short positions can be taken in stocks and bonds. However, even when such non-negative constraint is imposed, there is no essential difference in setting between them since their solutions conform to each other while the condition $w_1, w_2 \geq 0$ is satisfied.

[4] In setting the problem, the variance of the securities portfolio was used as amount of risk; however, actually, banks are supposed to perform risk assessment in reference to the quantiles in the loss distribution such as Value at Risk. However, since the variance of the portfolio is proportional to the quantiles when the changes in bond prices and stock prices follow the

maximum portfolio for a bank which is given exogenously. The value of the stock portfolio includes dividends. In contrast, the coupon of the bond portfolio is paid as a lump sum on the last day of the investment period.

Here, $B_{t,D}$ and S_t, which are the portfolio values of the bonds and the stocks, respectively, are assumed to take the following stochastic processes:

$$dS_t = \mu S_t dt + \sigma_s S_t dW_t^{(1)} \tag{4}$$

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r dW_t^{(2)} \tag{5}$$

$$dB_{t,D} = -D \cdot B_{t,D} dr_t \tag{6}$$

$$dW_t^{(1)} \cdot dW_t^{(2)} = \rho dt \tag{7}$$

where, $W_t^{(1)}$ and $W_t^{(2)}$ represent Brownian motion, r_t represents the interest rate during the period to maturity D ($D > 0$), μ represents the expected rate of return of the stocks including dividends, σ_s represents the volatility of the stock portfolio, κ represents the mean reversion speed of the interest rate, θ represents the mean reversion level of the interest rate, σ_r represents the interest rate volatility of the bond portfolio, and ρ represents the correlation coefficient of the Brownian motion. As shown in equation (6), for changes in the value of the bond portfolio, only the effect of the parallel shift in the yield curve is taken into consideration (the effect of changes in the slope or curvature of the yield curve is not considered)[5, 6]. Equation (5) was developed as a model to express fluctuations in the spot

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normal distribution, the problem setting by equation (1) can be considered to simulate a bank's actual behavior.

[5] If $B_{t,D}$ is a function of r_t, according to Ito's lemma, $dB_{t,D} = -D \cdot B_{t,D} dr_t + \frac{1}{2}\frac{\partial^2 B_{t,D}}{\partial r_t^2}\sigma_r^2 dt$. If we ignore the effect of second- and higher-order changes in the interest rate on the bond price, since $\partial^2 B_{t,D}/\partial r_t^2 = 0$, we obtain equation (6). If the interest rate changes significantly, the effect of second- and higher-order changes may not be ignored, but here, only the first-order changes are considered for convenience.

[6] Equation (6) only considers the parallel shift in the yield curve because when key rates in the number of $\{r_i\}_{i=1}^{I}$ and the key rate duration $\{D_i\}_{i=1}^{I}$ are considered, and if the bank invests ω_i only in bonds with the relevant duration, the average duration of the bond portfolio D is $D = \sum_{i=1}^{I} D_i \cdot \omega_i / \sum_{i=1}^{I} \omega_i$. On the other hand, if we ignore the effect of second- and higher-order changes in the interest rates on the bond price, the change in the value of the bond portfolio $B = \sum_{i=1}^{I} \omega_i$ is $dB = -\sum_{i=1}^{I} D_i \cdot \omega_i dr_i$. Therefore, if the yield curve makes a parallel shift, since

rates. Since the bond's duration is fixed in the analysis performed in this paper, the fluctuations in the government bond interest rates are expressed by such spot rate model[7].

2.2 Optimal asset holding ratio

Subject to the above assumptions, the solutions of equations (1) to (3) are as follows:

$$\begin{cases} w_1 = \frac{c-b-\sqrt{b^2-ac+\gamma(a-2b+c)}}{a-2b+c} \\ w_2 = \frac{a-b+\sqrt{b^2-ac+\gamma(a-2b+c)}}{a-2b+c} \end{cases} \tag{8}$$

However,

$$a = \mathbf{V}_0\left[B_{T,D}/B_{0,D}\right] = X^2Y^2(Y^2-1) \tag{9}$$

$$\begin{aligned} b &= \mathbf{Cov}_0\left[B_{T,D}/B_{0,D},\ S_T/S_0\right] \\ &= XYe^{\mu T}\left(\exp\left(\frac{-D\rho\,\sigma_s\,\sigma_r}{\kappa}(1-e^{-\kappa T})\right)-1\right) \end{aligned} \tag{10}$$

$$c = \mathbf{V}_0[S_T/S_0] = e^{2\mu T}\left(e^{\sigma_s^2 T}-1\right) \tag{11}$$

where,

$$X = \exp\left(-D(e^{-\kappa T}-1)(r_0-\theta)-\frac{1}{2}D^2\sigma_r^2 T\right) \tag{12}$$

$$Y = \exp\left(\frac{D^2\sigma_r^2}{4\kappa}(1-e^{-2\kappa T})\right) \tag{13}$$

(see the addendum for details of the derivation process). Therefore, if the parameters are determined, the optimal stockholding ratio can be uniquely elicited analytically by determining γ (the variance of the portfolio) from equation (8)[8].

$dr_i = dr\ (^\forall i)$, $dB = -D \cdot Bdr$ and thus the relationship in equation (6) is established.

[7] Given the movement in the interest rates shown in Figure 3, as mean reversion behaviors have been generally observed since 2000, it is considered to be appropriate to express the interest rate in the mean reversion process such as equation (5).

[8] The optimal holding ratio becomes an imaginary number and may not be elicited depending on the level of γ or volatility. This occurs when the amount of risk associated with the portfolio exceeds γ regardless of the setting of the stockholding ratio because the amount of existing bond holdings is too large compared to the volatility.

2.3 Parameters

Parameters are set using calibration and the maximum likelihood estimation. For particular parameters (σ_s, σ_r, and ρ), the values are set for two cases: the benchmark estimated by the samples since early 2000 and when the financial market is under stress[9]. In this way, it is possible to assess how the stockholding ratio changes in times of stress.

a. Calibration

Some parameters are set exogenously based on data, etc. The bank's investment period T is exogenously set at 1 year ($T = 1$ year). The duration of the bond portfolio D is set at 2.6 years and 3.9 years, respectively, for major banks and for regional banks based on the actual figures as of the end of March 2011. The coupon level of the bond portfolio is set so that it corresponds to the interest rate level of the currently issued bond whose maturity is close to the duration ($c_{0,D} = B_{0,D}r_0$). The expected rate of return μ of the stock portfolio is the average figure for the last 30 years ($\mu = 7.77\%$)[10]. For the correlation function ρ, the average correlation function estimated based on stock returns and the changes in interest rates since early 2000, 0.33, is used as the benchmark parameter. For the correlation function, the correlation coefficient at the time when stock prices and interest rates declined simultaneously after the bursting of the economic bubble, −0.63, is used as the parameter in times of stress (Figure 3).

b. Parameter estimation

First, from equation (5), the bond portfolio is assumed to follow the normal distribution where the mean and the variance of the interest rate are expressed as follows:

$$\mathbf{E}_0[r_t] = e^{-\kappa t}(r_0 - \theta) + \theta \tag{14}$$

$$\mathbf{V}_0[r_t] = \frac{\sigma_r^2}{2\kappa}(1 - e^{-2\kappa t}) \tag{15}$$

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[9] The effect of $\mu, \kappa,$and θ on the stockholding ratio is not much different between the benchmark and in times of stress compared to the effect of the volatility and the correlation.

[10] Since the average stock return calculated for the past 10 years takes a negative value, the solution of the optimization fails to describe the bank's realistic investment behavior. This analysis adopted a long-term average for the last 30 years given that in fact, the stock return assumed by investors should be higher than this level.

If the number of samples is N, the log likelihood of the bond portfolio L_B is expressed as:

$$L_B = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln\left(\frac{\sigma_r^2}{2\kappa}(1-e^{-2\kappa\tau})\right) - \frac{\kappa}{\sigma_r^2(1-e^{-2\kappa\tau})}\sum_{i=1}^{N}(r_{i\tau} - e^{-\kappa\tau}(r_0-\theta)+\theta)^2 \tag{16}$$

Here, τ is the sample time interval and is set as $\tau = 1/250$. κ, θ, and σ_r are estimated to be the parameters which maximize L_B.

The stock portfolio is assumed to follow the log-normal distribution where the mean and the variance are expressed as follows from equation (4):

$$\mathbf{E}_0\left[\ln\left(\frac{S_t}{S_0}\right)\right] = \left(\mu t - \frac{\sigma_s^2 t}{2}\right) \tag{17}$$

$$\mathbf{V}_0\left[\ln\left(\frac{S_t}{S_0}\right)\right] = \sigma_s^2 t \tag{18}$$

The log likelihood of the stock portfolio L_S is expressed as:

$$L_S = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma_s^2\tau) - \frac{1}{2\sigma_s^2\tau}\sum_{i=1}^{N}\left(\ln\left(\frac{S_{i\tau}}{S_{i\tau-1}}\right) - \left(\mu\tau - \frac{\sigma_s^2\tau}{2}\right)\right)^2 \tag{19}$$

Here, the partial differential value for σ_s and μ is zero:

$$\mu = \frac{1}{N\tau}\sum_{i=1}^{N}\ln\left(\frac{S_{i\tau}}{S_{i\tau-1}}\right) + \frac{\sigma_s^2}{2} \tag{20}$$

$$\sigma_s^2 = \frac{1}{N\tau}\sum_{i=1}^{N}\left(\ln\left(\frac{S_{i\tau}}{S_{i\tau-1}}\right) - \frac{1}{N}\sum_{i=1}^{N}\left(\ln\frac{S_{i\tau}}{S_{i\tau-1}}\right)\right)^2 \tag{21}$$

(see Table 1 for basic statistics).

The estimation uses the composite index of the interest rates of currently issued 3-year bonds calculated by Bloomberg as the data for the interest rate r_t (see Table 1 for the basic statistics of the samples used for the estimation). TOPIX is used as the market portfolio data of stock prices.

Of the parameters, θ, κ, σ_s and σ_r are estimated based on the daily data from early 2000 to the end of November 2011. In particular, for σ_s, σ_r and ρ parameters are estimated for two cases: the benchmark and the times of stress. The estimation results based on all samples are used as the estimates for the benchmark parameters. For the parameters for the times of stress, estimation is performed with a 1-year rolling window and the estimation results at the time when volatilities σ_s and σ_r are the greatest and at the time when the correlation coefficient ρ is the smallest are used as the respective estimates.

Table 2 shows the estimation results. The mean reversion level of the interest rate θ and the mean reversion speed of the interest rate κ are estimated to be 0.45% and 0.52, respectively. The stock price volatility σ_s is estimated to be 23.1% for the benchmark. For the parameters in times of stress, the stock price volatility increases to 42.4% at the time of the Lehman Shock. The interest rate volatility σ_r is estimated to be 0.30% for the benchmark and the estimate rises to 0.49% at the time of the Lehman Shock.

3. Assessment for the Whole Banking Sector

3.1 The stockholding ratio and the variance of the benchmark

Figure 5 shows the relation between the stockholding ratio and the variance of the total portfolio of major and regional banks under the benchmark parameters. The vertical and horizontal axes represent the variance $\sqrt{\gamma}$ and the optimal stockholding ratio w_2^*, respectively.

From the results, firstly, we can see that there is a non-linearity between the stockholding ratio and the variance. This is because the variance $\sqrt{\gamma}$ and the stockholding ratio w_2^* obtained analytically in equation (8) have a binominal series relation. As the stockholding ratio rises, the variance of the bank's portfolio becomes larger at an accelerated pace. Secondly, since the average duration of bond investments is longer and the amount of risk associated with bond holding is larger for regional banks, if the stockholding ratio is low, the variance of regional banks' portfolios is larger than that of major banks. Thirdly, when the stockholding ratio rises to about 10%, the difference in the portfolio variance between major and regional banks is reduced. As the stockholding increases, the bondholding decreases because the amount of equity capital which can be allocated to the amount of interest rate risk associated with the bondholding is reduced. Therefore, for regional banks in particular, the variance associated with bond investment is more significantly reduced. At present (the end of the first half of FY2011), since the stockholding ratios of major banks are higher than those of regional banks, the amount of risk of the securities portfolio is larger for major banks.

3.2 Portfolio variance and the amount of risk in financial institutions

So far, we have explored the relation between the stockholding ratio and the variance of the total securities portfolio of financial institutions. Incidentally, if we use the portfolio variance value, we should be able to calculate the amount of loss suffered by a financial institution based on certain assumptions, which is typically VaR. If we assume that the rate of return of the portfolio follows the normal distribution, the amount of loss which may be suffered by a financial institution with a fixed probability can be calculated by giving the variance value to VaR. This is also the amount of risk held by the financial institution. Specifically, this can be calculated by multiplying the volatility by a constant and then multiplying by the portfolio value. Here, since 99%VaR is assumed as the amount of risk, it is multiplied by 2.33, which is 1 percentile of one side of the normal distribution. Based on these assumptions, subject to the current stockholding ratio, major and regional banks would suffer losses exceeding 5 trillion yen and 2 trillion yen, respectively, with a probability of 1% (the vertical line in the figure represents the stockholding ratio as of the end of the first half of FY 2011).

3.3 Effect of different financial market conditions on the stockholding ratio and the amount of risk

This section analyzes the effect of different financial market conditions on the stockholding ratio and the amount of risk. Here, three situations are assumed as the financial market conditions: where the correlation coefficient between the interest rate and the stock return ρ declines, where the stock price volatility σ_s increases, and where the interest rate volatility σ_r increases. Figures 7 to 12 show the total amount of risk given the stockholding ratio at the present time when the correlation coefficient between the interest rate and the stock return, the stock price volatility and the interest rate volatility are changed continuously.

Figures 7 and 8 show the amount of risk for different correlation coefficients. When the correlation coefficient moves in a negative direction, since the effect of holding stocks to hedge the bond losses is lost and losses arise simultaneously from the stocks and bonds, the amount of risk should increase. For example, since the hedging effect is lost when the correlation coefficient becomes zero, the amount of risk increases by approx. 0.6 trillion yen and 0.5 trillion yen for major and regional banks, respectively, compared to the benchmark case. On the other hand, if the correlation coefficient reverses to −0.63 as seen in times of crisis, since the stock and bond portfolios move in the same direction, the amount of risk increases by approx. 1.6 trillion yen and 1.2 trillion yen for major and regional banks, respectively compared to the benchmark case.

Figures 9 and 10 show the amount of risk when the stock price volatility rises. The higher the volatility, the larger the amount of risk associated with stockholding would become, thus increasing the total amount of risk. For example, if the stock price volatility rises to the level at the time of the Lehman Shock, the amount of risk increases by approx. 5 trillion yen and 2 trillion yen for major and regional banks, respectively, compared to the benchmark case.

Figures 11 and 12 show the amount of risk when the interest rate volatility rises. Since the higher the volatility, the larger the amount of risk associated with bonds, the total amount of risk is increased. However, the size is smaller compared to the case of stock price volatility. If the interest rate volatility rises to the level at the time of the Lehman Shock under stress, the amount of risk increases by approx. 0.1 trillion yen and 0.5 trillion yen for major and regional banks, respectively, compared to the benchmark case.

From the above results, we can see that in Japan, the correlation coefficient between the interest rate and stock prices and the stock price volatility play more important roles than the interest rate volatility in determining the bank's asset composition ratio. This implies that banks should reduce their stockholdings because of their large amount of risk.

4. Assessment for Individual Financial Institutions

4.1 Assessment of the benchmark

This section compares the stockholding ratio calculated from the model and the actual stockholding ratio of individual financial institutions according to their equity capital requirements. Here, the amount of equity capital which can be allocated to bonds and stocks

(= capital buffer) is defined as Tier1 capital less the amount of regulatory equity capital and the credit risk, the operational risk and the foreign bond holding risk[11, 12]. This is because core capital is considered to be primarily used to absorb the risk associated with lending, which is banks' main line of business. The amount of the foreign bond holding risk was subtracted in advance since such risk is not considered in this model.

In Figure 13, the horizontal axis represents the difference between the allowable stockholding ratio according to the capital buffer calculated from the model and the actual stockholding ratio and the vertical axis represents the number of banks. The banks showing negative figures hold more stocks than the level allowed by the capital buffer. Overall, about 20% of the banks hold excessive stocks relative to the capital buffer. In addition, both among major and regional banks, some banks' stockholding ratios are quite high and certain banks' stockholding ratios are excessive and exceed 5% of the capital buffer.

These results show that even in case of the benchmark, some banks hold greater equity risk than the allowable amount of risk and such banks need to further reduce their stockholdings.

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[11] The amount of regulatory equity capital is set at 8% and 4% of the risk assets for international and domestic reference banks, respectively. The amount of the credit risk is assumed to be the non-expected loss at the confidence level of 99%, and is estimated based on the default probability calculated from the borrower classification data for bank lending and the collection rate of bank loans in the event of loss. The amount of the foreign bond holding risk was calculated by multiplying VaR of foreign interest rates at the confidence level of 99% by the average maturity of bonds. Here, VaR of foreign interest rates was obtained by weighted-averaging VaR of 3-year government bond interest rates in each currency based on the Japanese banks' outstanding bond balances by country estimated by the balance of payments statistics. The amount of operational is set at 15% of the gross profit.

[12] This paper subtracted the amount of the regulatory equity capital requirement in calculating the capital buffer to be allocated to bonds and stocks, taking the bank's reputational risk into consideration. Should heavy losses arise from a bank's securities portfolio and the bank's equity capital ratio falls below the regulatory requirement level, prompt corrective action is implemented on the bank by the supervisory authorities. If, as a result, credit uncertainty spreads about the bank, it may trigger a situation such as a run on the bank, illustrating the bank's reputational risk. Therefore, though the regulatory equity capital in essence is the stock of loss absorption potential to provide for the occurrence of a tail risk, in reality, it is important not to fall below the regulatory standard even in time of crisis involving significant deterioration of the financial environment.

4.2 Effect of different financial market conditions

Next we compare the stockholding ratio allowed based on the capital buffer and the actual stockholding ratio when the financial market conditions change and the correlation coefficient between the interest rate and stock return ρ declines, the stock price volatility σ_s rises and the interest rate volatility σ_r rises compared to the benchmark conditions.

Figures 14, 15 and 16 show the difference between the allowable stockholding ratio subject to the capital buffer based on each parameter and the actual stockholding ratio for the correlation coefficient, the stock price volatility and the interest rate volatility, respectively. In all cases, more banks hold excessive stocks relative to the capital buffer compared to the benchmark case (Figure 13). This result implies that both major and regional banks need to reduce their stockholdings significantly given the possibility of market environment deterioration. In particular, for major banks, the stockholding ratio of most of the banks becomes excessive in the scenario where the stock price volatility increases, reflecting the fact that the near-term level of the stockholdings is high. For regional banks, the stockholding ratio of most of the banks becomes excessive in the scenario where the correlation coefficient between the interest rate and stock prices is reversed, reflecting the fact that the durations of bonds are longer than those held by major banks.

4.3 Issues to be considered

There are several issues to be considered about the analysis results so far in relation to the assumptions. First is the amount of capital which serves as a constraint. In the above calculation, we assumed that the amount of equity capital less the equity capital to cover the lending risk and the regulatory equity capital is allocated to the securities portfolio. Under such assumption, the stockholding ratio is calculated conservatively. If the regulatory equity capital is allocated to the amount of risk of the securities portfolio without subtraction, as a matter of course, the ratio of stocks held rises as well. However, if the losses from the securities portfolio actually erodes the regulatory equity capital due, for example, to a stock price decline, this may cause difficulties in normal business operations as a result of deterioration of the reputation in the market and financing problems. In the above calculation, we used a conservative assumption for the amount of risk allocated to securities taking these factors into consideration.

The second is the relationship with the financial institution's risk management. When a financial institution controls its risk, it first calculates the potential losses incurred at the time of stress and considers whether the sum total of such losses remains within the range of the capital. For the purpose of the above analysis, this is the same as calculating the amount of risk based on the assumptions that the correlation between asset classes is the highest and that stocks and bonds suffer maximum losses at the same time. Furthermore, the amount of risk is assessed inclusive of the regulatory capital. Accordingly, as mentioned earlier, the analysis in this paper is different from the risk management method of an ordinary financial institution in that it assumes conservative capital use as it first subtracts the amount of regulatory capital and then measures the level of stocks available for holding relative to the remaining capital.

The third is the assumption about the stock return. The analysis in this paper used the average value over the last 30 years (7.77%) as the stock return. However, if we calculate the latest return (over the last 10 years), the average value becomes negative and the expected dividend rate (as of the end of March 2011) is also at a quite low level of 1.66%. Therefore, if we use the latest return, the stockholding ratio becomes even lower. On the other hand, it is also important how to take into consideration the advantages from other banking activities obtained as strategic stockholdings in addition to direct returns. Some consider that the benefit of earning commission income and loan margins from a company stably for a long time by holding the company's stock for a long time should be added to the direct return obtained from the stocks. If such return is actually measurable, it would be possible to add such return to the direct return to calculate the total return, which would then be used to work out the optimal stockholding ratio.

5. Conclusions and Future Issues

This paper calculated the optimal asset composition ratio using the portfolio model taking into consideration the correlation between the equity risk and the interest rate risk. Within the range of variations in the parameters since the 1990s, it was identified that fluctuations in the stock price volatility and the correlation coefficient of stock prices and the interest rate had more impact on the stockholding ratio than the interest rate volatility. We also found that some banks held more stocks than the allowable level based on the capital buffer. We concluded that, when the financial market is under stress, the percentage of such banks will increase.

Lastly, we suggest two directions to further develop this paper. The first is to analyze the lending, which is a bank's core business, with consideration of the correlation with other asset classes. The credit risk has the same distribution of losses and profits as stocks and bonds and needs to be analyzed together with the other two types of risks. The second issue is the necessity of expanding the analysis to a dynamic problem since a bank's risk management method cannot be expressed properly only by solving the static optimization problem. In fact, stock prices and interest rates vary over time and risk hedging behaviors should be taken accordingly. It is necessary to deepen the analysis to know how the results of our analysis would be changed by factoring in these behaviors. It is also necessary to pay attention to the sample period for measuring the return and risk of assets.

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Addendum. Derivation of Optimal Stockholding / Bondholding Ratio

1. Optimization problem

Given equation (3), equation (1) is:

$$\max_{\{w_2\}}\left[w_2\left(\mathbf{E}_0\left[\frac{S_T}{S_0}\right]-\mathbf{E}_0\left[\frac{B_{T,D}+c_{0,D}T}{B_{0,D}}\right]\right)+\mathbf{E}_0\left[\frac{B_{T,D}+c_{0,D}T}{B_{0,D}}\right]-1\right], \tag{22}$$

Therefore, if the expected rate of return of the stock portfolio exceeds that of the bond portfolio, that is, if the value in the brackets of the above equation is positive, from conditional equation (2), the solution of equation (22) is the larger of the solutions of the second-degree equation (24) as described below. If the variances of $B_{T,D}/B_{0,D}$ and S_T/S_0 are expressed as a and c respectively, and the covariance as b, then

$$\mathbf{V}_0[\phi_t]=\boldsymbol{w}^T\begin{pmatrix}a & b\\ b & c\end{pmatrix}\boldsymbol{w}=\gamma \tag{23}$$

$$\Leftrightarrow\ (a-2b+c)w_2^2+2(b-a)w_2+a-\gamma=0. \tag{24}$$

Since $a-2b+c>0$, the solution of the above equation is equation (8).

The value in the brackets of equation (22) calculated from the parameters set in this paper is positive mainly because the stock dividends exceed the interest rate of 3-year government bonds[13].

2. Variance and Covariance of Stock and Bond Subportfolios

Next, the variances a and c and the covariance b of the value of the bond and stock subportfolios, $B_{t,D}/B_{0,D}$, respectively, are obtained.

First, the variance of the bond portfolio a is derived. From equation (5):

$$r_t=e^{-\kappa t}(r_0-\theta)+\theta+\sigma_r\int_0^t e^{\kappa(s-t)}\,dW_s^{(2)}.$$

If the values from 0 to t are integrated with respect to time:

[13] This paper assumes that the expected rate of return on stock prices excluding dividends μ is zero. If the expected rate of return on stocks is negative, the condition may not be satisfied. In such a case, the optimal stockholding ratio may be zero or negative (it is optimal to take short positions on stocks).

$$\int_0^t r_s\, ds = \frac{1-e^{-\kappa t}}{\kappa}(r_0-\theta) + \theta t - \frac{\sigma_r}{\kappa}\int_0^t \left(e^{-\kappa(t-u)}-1\right) dW_u{}^{(2)} \tag{25}$$

On the other hand, from equations (5) and (6):

$$d\ln B_{t,D} = -D\kappa(\theta - r_t)dt - D\sigma_r dW_t{}^{(2)} - \frac{1}{2}D^2\sigma_r^2 dt.$$

If the values from 0 to t are integrated with respect to time:

$$\ln\frac{B_{t,D}}{B_{0,D}} = -D\kappa\theta t - \frac{1}{2}D^2\sigma_r^2 t + D\kappa\int_0^t r_s\, ds - D\sigma_r\int_0^t dW_s{}^{(2)}.$$

Substituting into equation (25) and rearranging:

$$\begin{aligned}\frac{B_{t,D}}{B_{0,D}} &= \exp\left(-\frac{1}{2}D^2\sigma_r^2 t - D(e^{-\kappa t}-1)(r_0-\theta) - D\sigma_r\int_0^t e^{-\kappa(t-s)}dW_s{}^{(2)}\right)\\ &= X\cdot\exp\left(-D\sigma_r\int_0^t e^{-\kappa(t-s)}dW_s{}^{(2)}\right).\end{aligned} \tag{26}$$

X is defined by equation (12) (Here and hereafter we describe the investment horizon T as t). If Y, which is defined by equation (13) is used, then:

$$\mathbf{E}_0\left[\exp\left(-D\sigma_r\int_0^t e^{-\kappa(t-s)}dW_s{}^{(2)}\right)\right] = \exp\left(\frac{D^2\sigma_r^2}{4\kappa}(1-e^{-2\kappa t})\right) = Y,$$

so the mean of the bond portfolio as the expected value of equation (26) is:

$$\mathbf{E}_0\left[\frac{B_{t,D}}{B_{0,D}}\right] = XY, \tag{27}$$

Since

$$\mathbf{E}_0\left[\left(\frac{B_{t,D}}{B_{0,D}}\right)^2\right] = X^2\mathbf{E}_0\left[\exp\left(-2D\sigma_r\int_0^t e^{-\kappa(t-s)}dW_s{}^{(2)}\right)\right] = X^2Y^4, \tag{28}$$

from equations (27) and (28), the variance of the bond portfolio is:

$$a = \mathbf{V}_0\left[\frac{B_{t,D}}{B_{0,D}}\right] = X^2Y^2(Y^2-1), \tag{29}$$

Now, we derive the variance of the stock portfolio c. From equation (4):

$$\frac{S_t}{S_0} = \exp\left(\mu t - \frac{\sigma_s^2 t}{2} + \sigma_s \int_0^t dW_s{}^{(1)}\right), \tag{30}$$

so

$$\mathbf{E}_0\left[\frac{S_t}{S_0}\right] = e^{\mu t}, \qquad \mathbf{E}_0\left[\left(\frac{S_t}{S_0}\right)^2\right] = e^{\left(2\mu t + \sigma_s^2 t\right)}, \tag{31}$$

Therefore, c is:

$$c = \mathbf{V}_0\left[\frac{S_t}{S_0}\right] = e^{2\mu t}\left(e^{\sigma_s^2 t} - 1\right), \tag{32}$$

Lastly, we derive the covariance b. From equations (26) and (30):

$$\mathbf{E}_0\left[\frac{B_{t,D}}{B_{0,D}} \cdot \frac{S_t}{S_0}\right] =\cdot \exp\left(\mu t - \frac{\sigma_s^2 t}{2}\right) \mathbf{E}_0\left[\exp\left(-D\sigma_r \int_0^t e^{-\kappa(t-s)} dW_s{}^{(2)} + \sigma_S \int_0^t dW_s{}^{(1)}\right)\right]. \tag{33}$$

where, if

$$\alpha_t = -D\sigma_r \int_0^t e^{-\kappa(t-s)} dW_s{}^{(2)}, \quad \beta_t = \sigma_s \int_0^t dW_s{}^{(1)},$$

then:

$$\mathbf{E}_0[\alpha_t] = 0, \ \ \mathbf{E}_0[\beta_t] = 0, \ \ \mathbf{V}_0[\alpha_t] = \frac{D^2\sigma_r^2}{2\kappa}\left(1 - e^{-2\kappa t}\right), \ \ \mathbf{V}_0[\beta_t] = \sigma_s^2 t$$

Since it is calculated as:

$$\mathbf{Cov}_0[\alpha_t, \beta_t] = -D\rho\sigma_r\sigma_s\left(\frac{1 - e^{-\kappa t}}{\kappa}\right),$$

$$\begin{aligned}\mathbf{E}_0\left[\frac{B_{t,D}}{B_{0,D}} \cdot \frac{S_t}{S_0}\right] &= X \cdot \exp\left(\mu t - \frac{\sigma_s^2 t}{2}\right) \exp\left(\mathbf{E}_0[\alpha_t + \beta_t] + \frac{1}{2}\mathbf{V}_0[\alpha_t] + \frac{1}{2}\mathbf{V}_0[\beta_t] + \mathbf{Cov}_0[\alpha_t, \beta_t]\right) \\ &= XYe^{\mu t} \exp\left(-D\rho\sigma_r\sigma_S \frac{1 - e^{-\kappa t}}{\kappa}\right),\end{aligned} \tag{34}$$

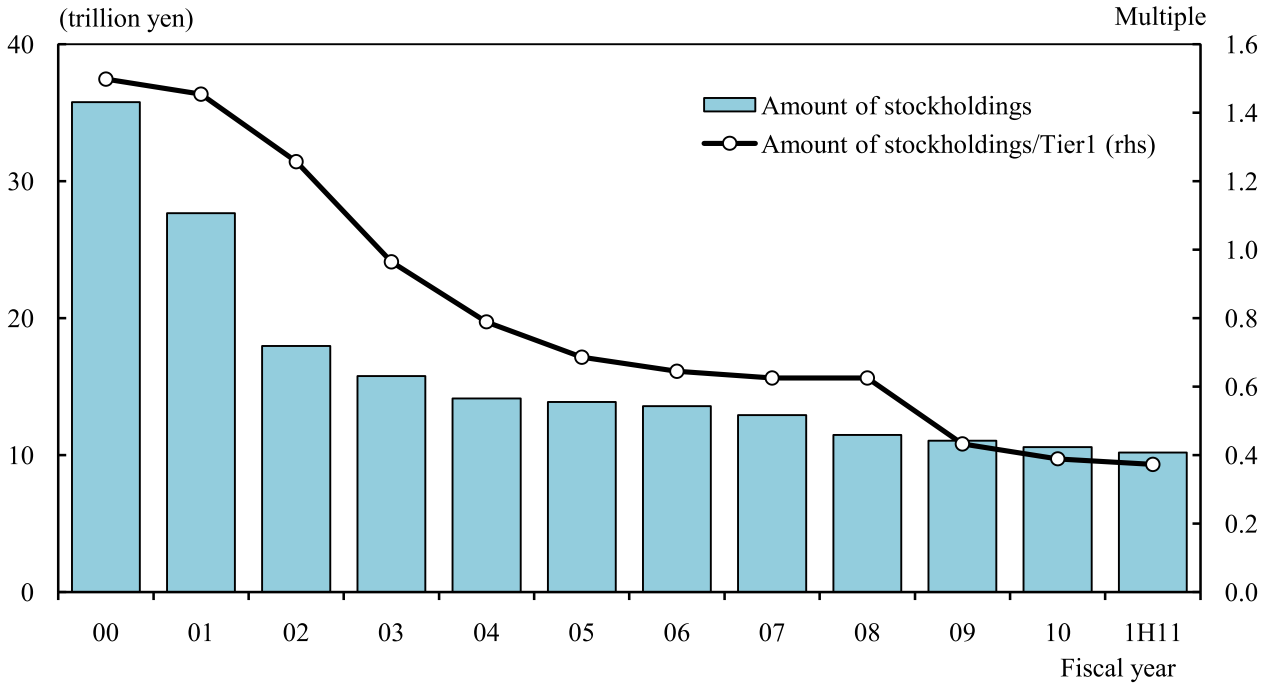
From equations (27), (31) and (34):

$$b = \mathbf{Cov}_0\left[\frac{B_{t,D}}{B_{0,D}}, \frac{S_t}{S_0}\right] = XYe^{\mu t}\left(\exp\left(-D\rho\sigma_r\sigma_s\frac{1-e^{-\kappa t}}{\kappa}\right) - 1\right). \tag{35}$$

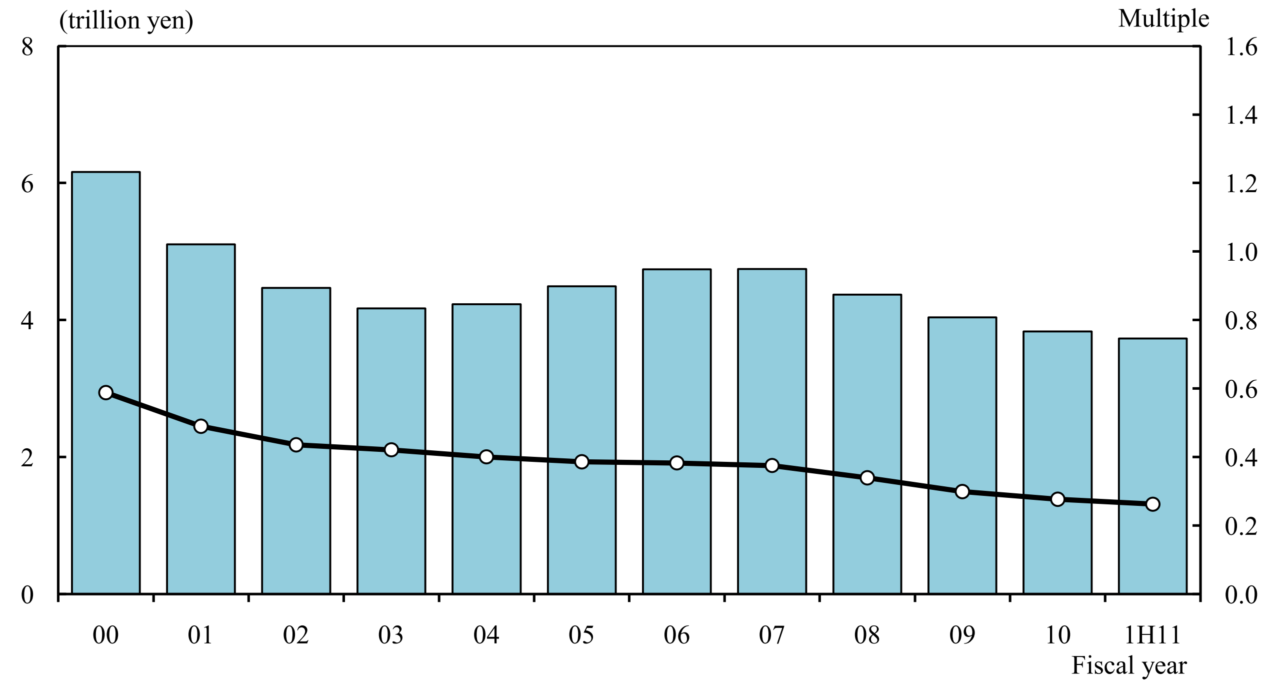
(Figure 1)

Outstanding Amount of Stockholdings

(a) Major banks



(b) Regional banks

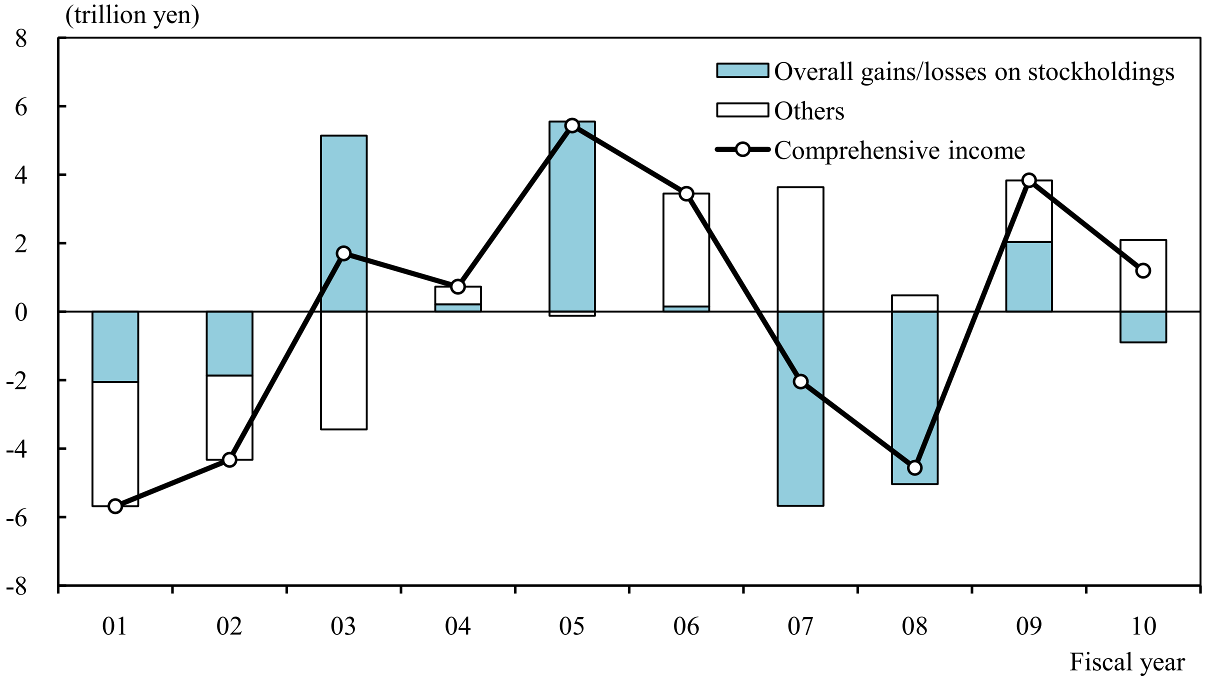


Note: excluding subsidiaries and affiliates.
Source: Bank of Japan

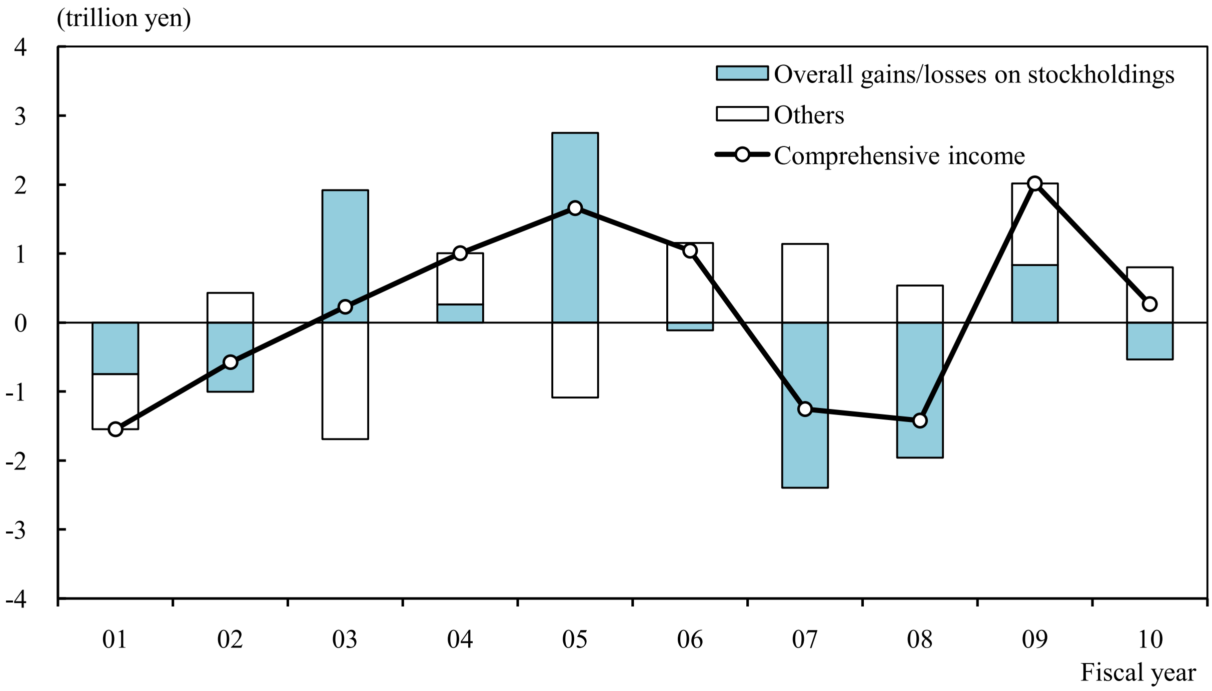
(Figure 2)

Comprehensive Income

(a) Major banks



(b) Regional banks

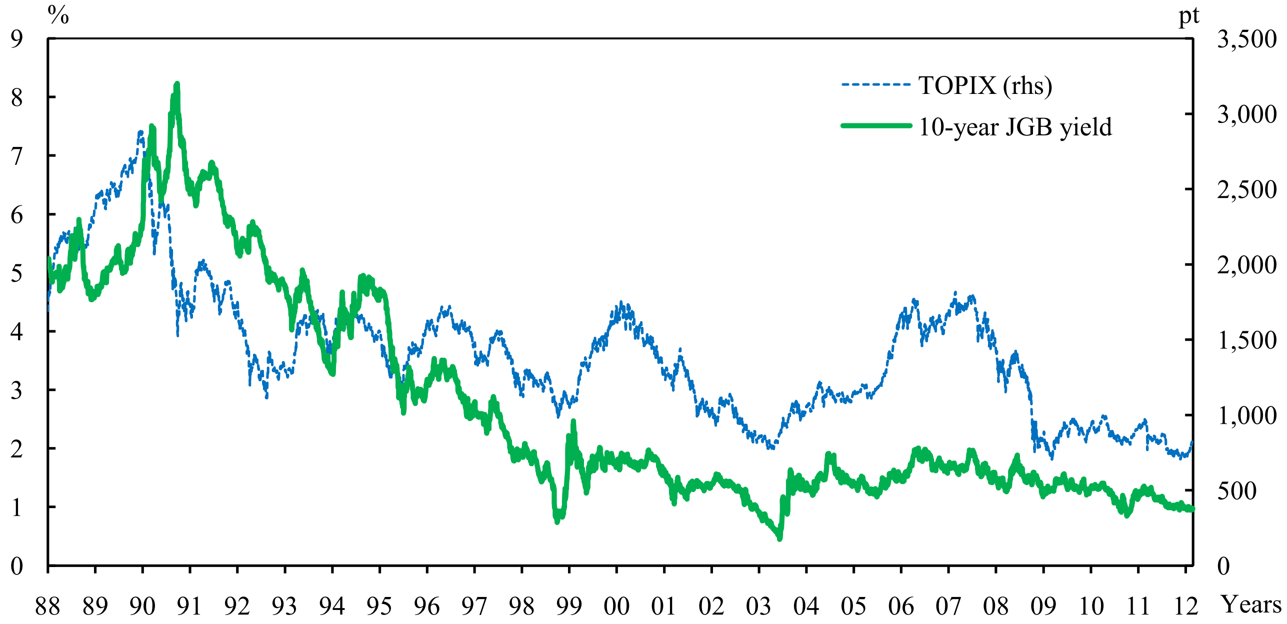


Note: Overall gains/losses on stockholdings are the sum of realized gains/losses multiplied by 0.6 and changes in unrealized gains/losses on stockholdings.
Source: Bank of Japan

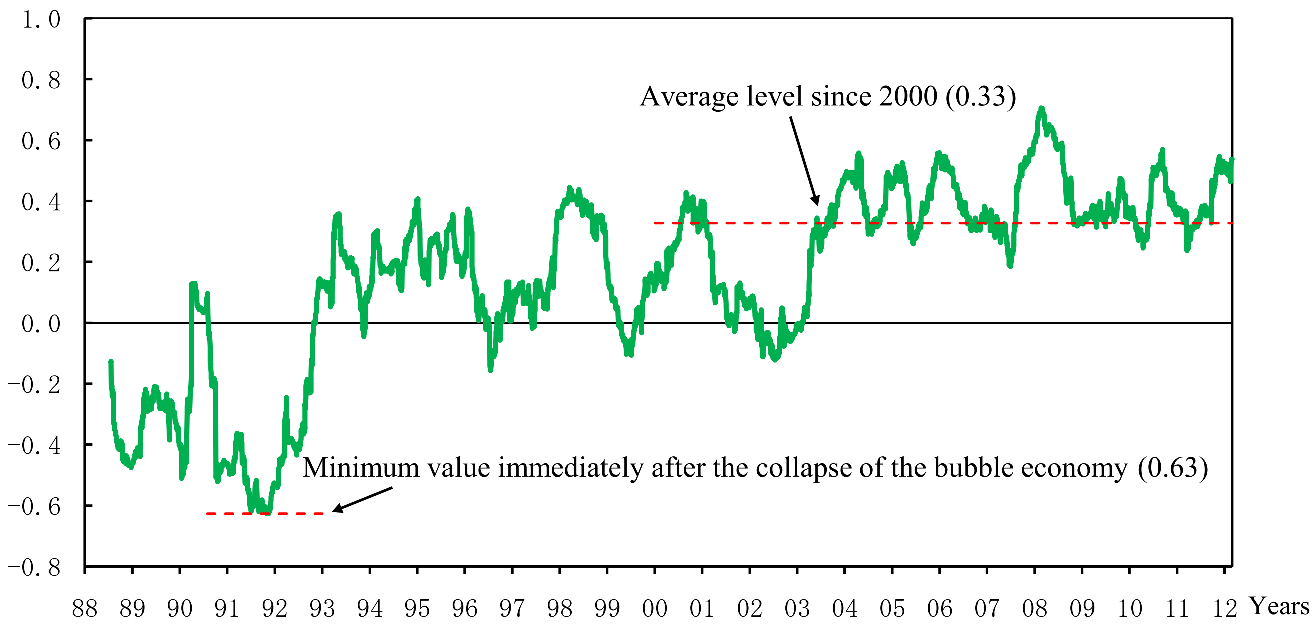
(Figure 3)

Correlation between Stock Prices and Interest Rates in Japan

(a) Stock prices and interest rates



(b) Correlation coefficient between stock prices and interest rates

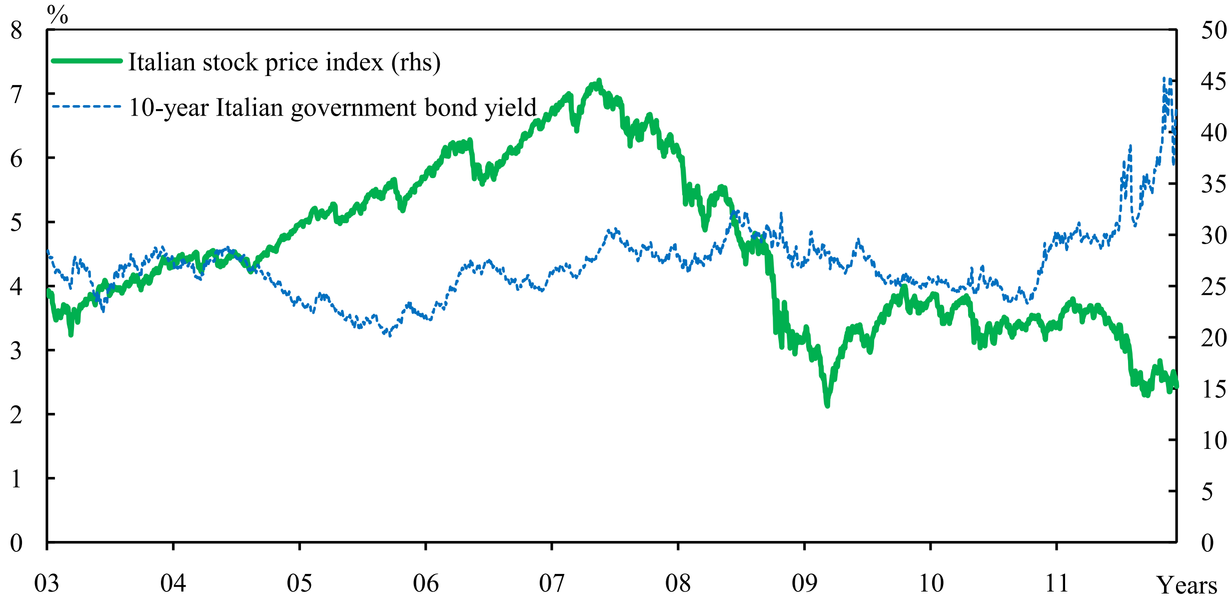


Note: Correlation coefficients are calculated based on daily returns on TOPIX and daily changes in 10-year government bond interest rates durning a 130-day rolling window.
Source: Bloomberg

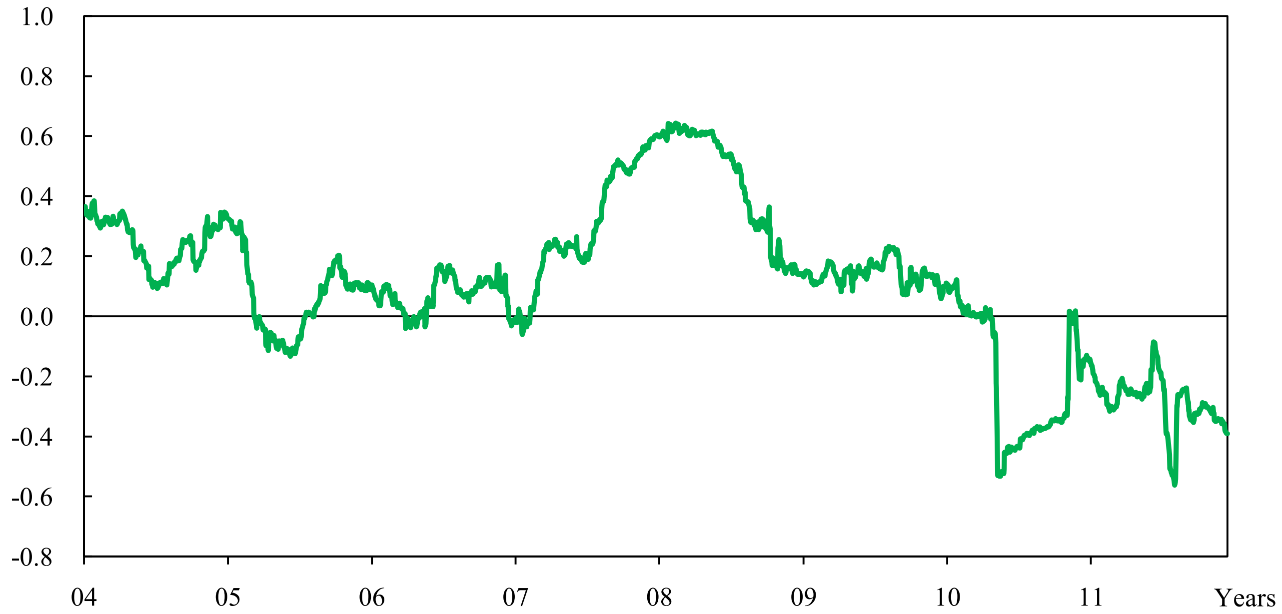
(Figure 4)

Correlation between Stock Prices and Interest Rates in Italy

(a) Stock prices and interest rates



(b) Correlation coefficient between stock prices and interest rates

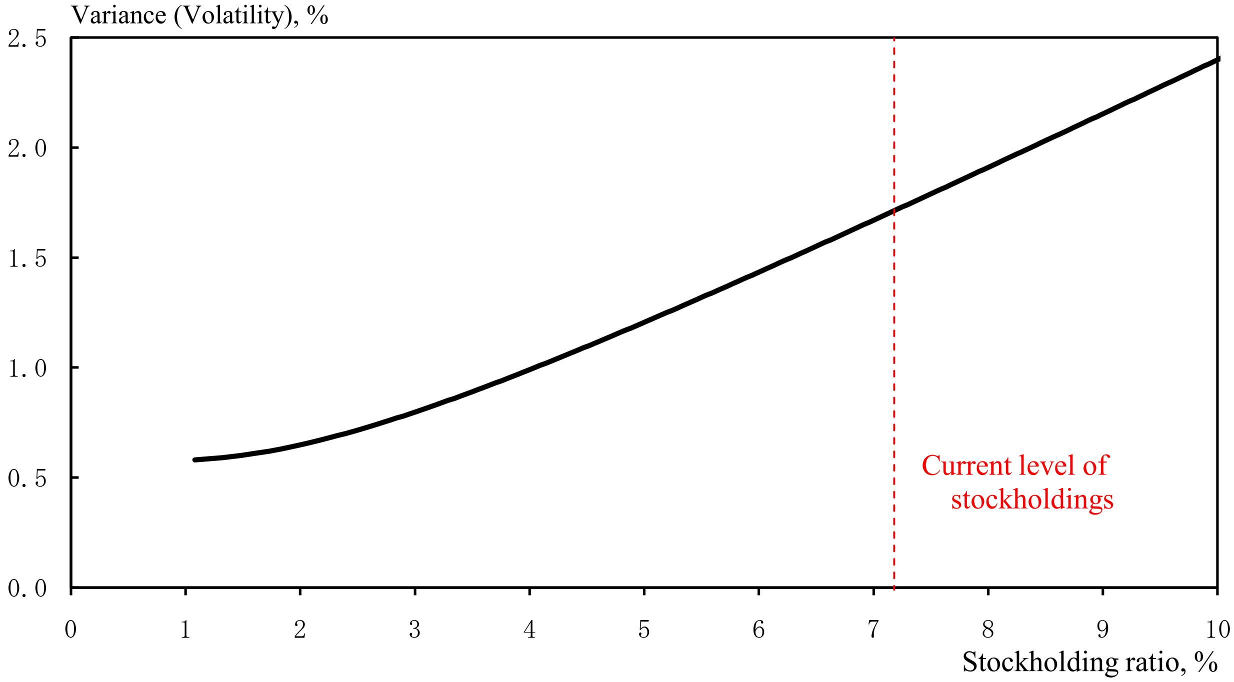


Note: Correlation coefficients were calculated based on daily returns on the Italian stock price index and daily changes in 10-year government bond interest rates during a 130-day rolling window.
Source: Bloomberg

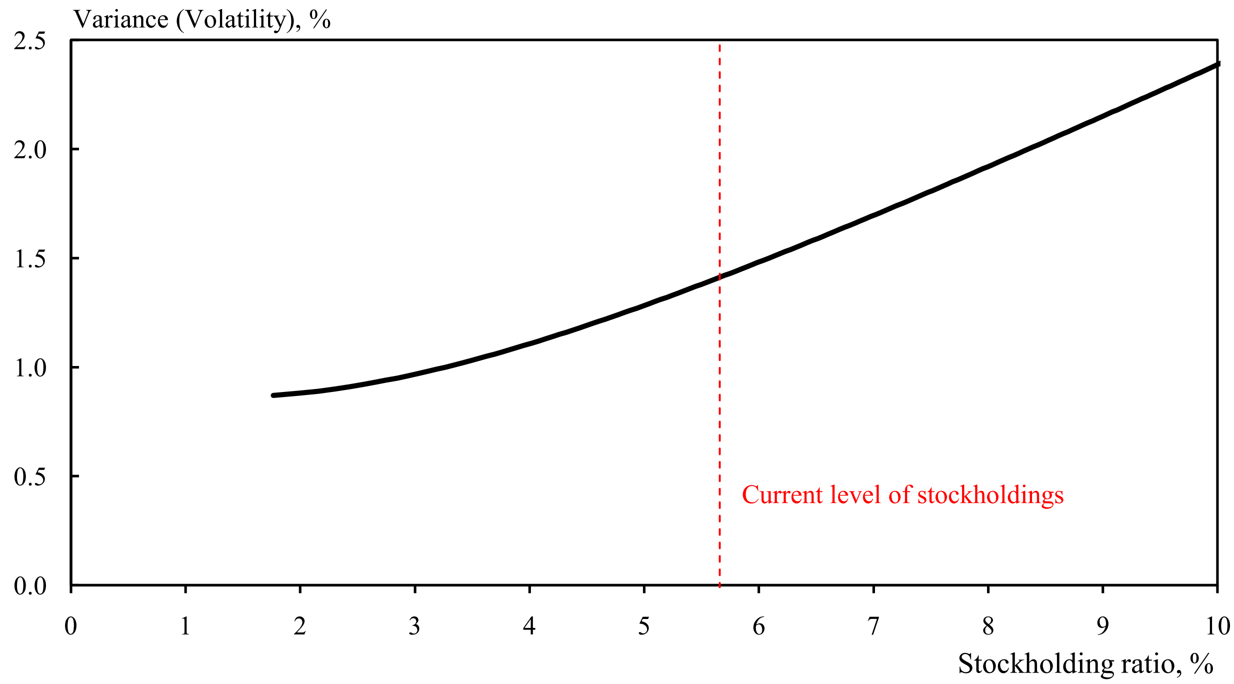
(Figure 5)

Variance of Portfolio and Optimal Stockholding Ratio

(a) Major banks



(b) Regional banks

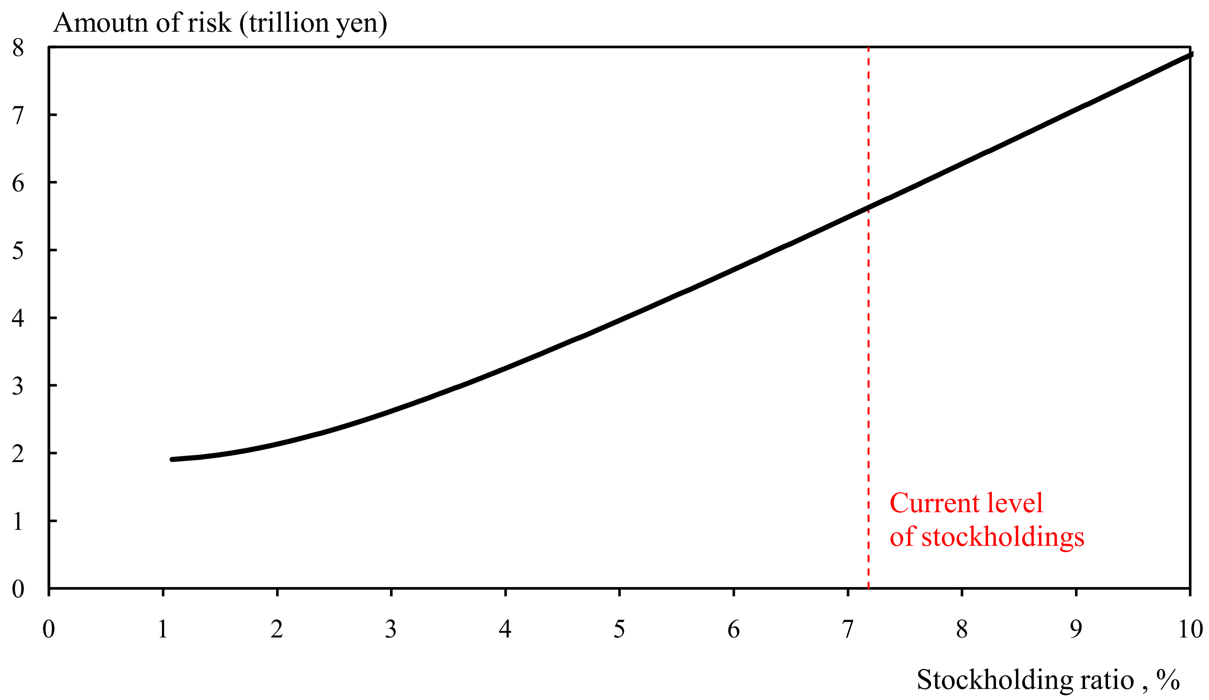


Note 1: Duration *D* of the bond portfolio is the actual value for major and regional banks as of the end of FY2010.
Note 2: The current level of stockholdings is the actual figure as of the end of the first half of FY2011.

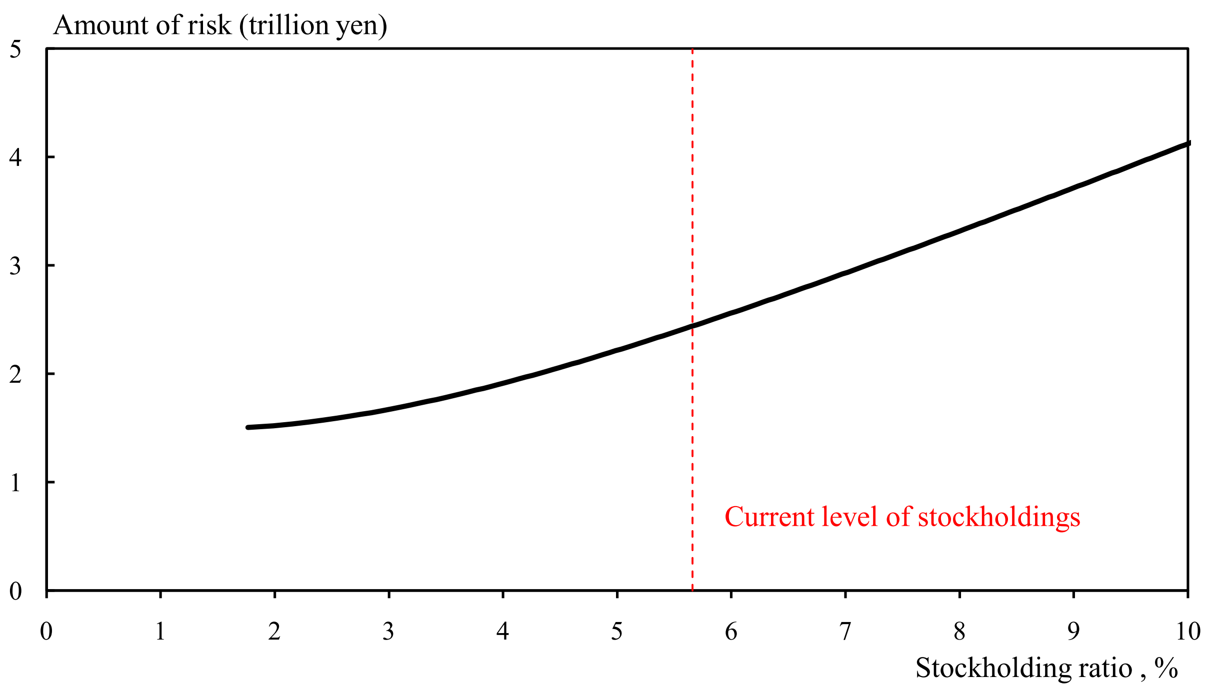
(Figure 6)

Amount of Losses (Risk) when a 1% Shock Occurs and Optimal Stockholding Ratio

(a) Major banks



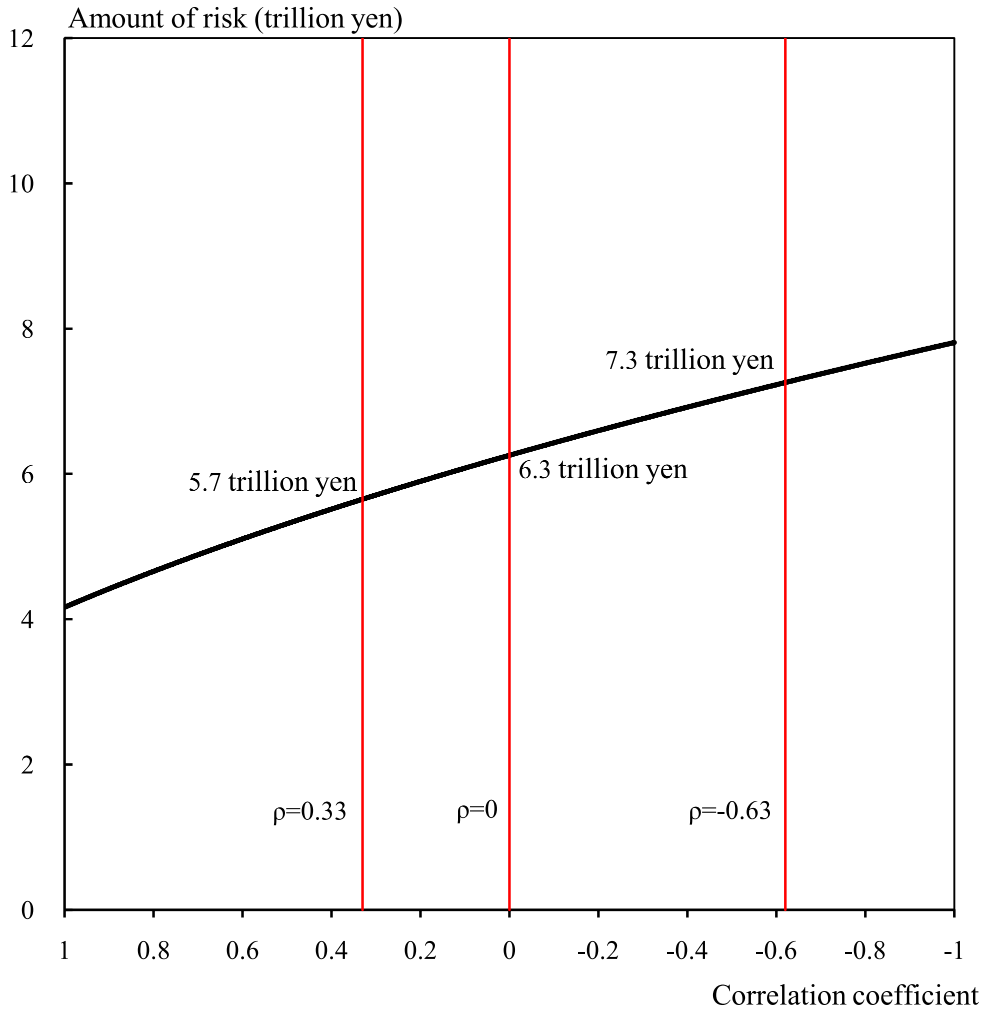
(b) Regional banks



Note 1: Duration D of the bond portfolio is the actual value for major and regional banks as of the end of FY2010.
Note 2: The current level of Stockholding ratio is the actual figure as of the end of the first half of FY2011.

(Figure 7)

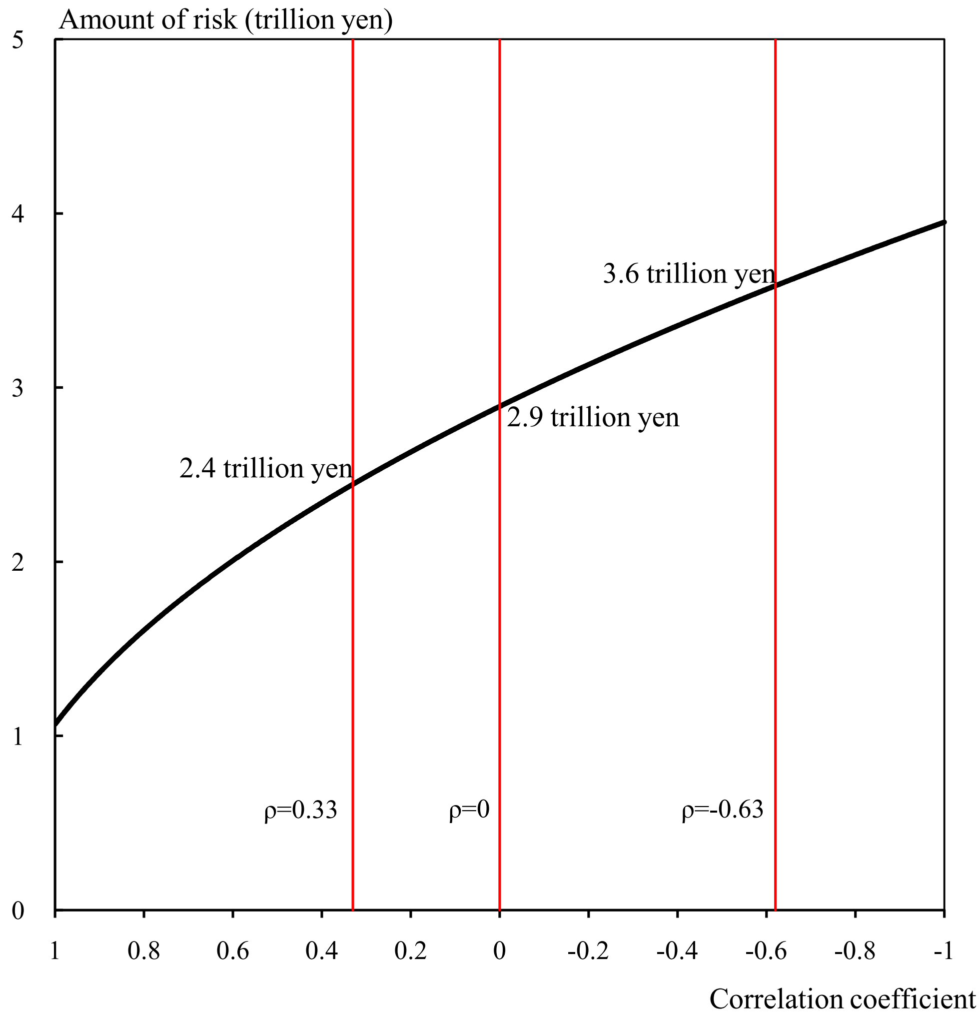
Changes in Correlation Coefficients and Amount of Risk (Major Banks)



Note: The vertical lines, from left to right, represent the average level since 2000 (= 0.33), no correlation (= 0) and the realized value after the collapse of the bubble economy (= −0.63), respectively.

(Figure 8)

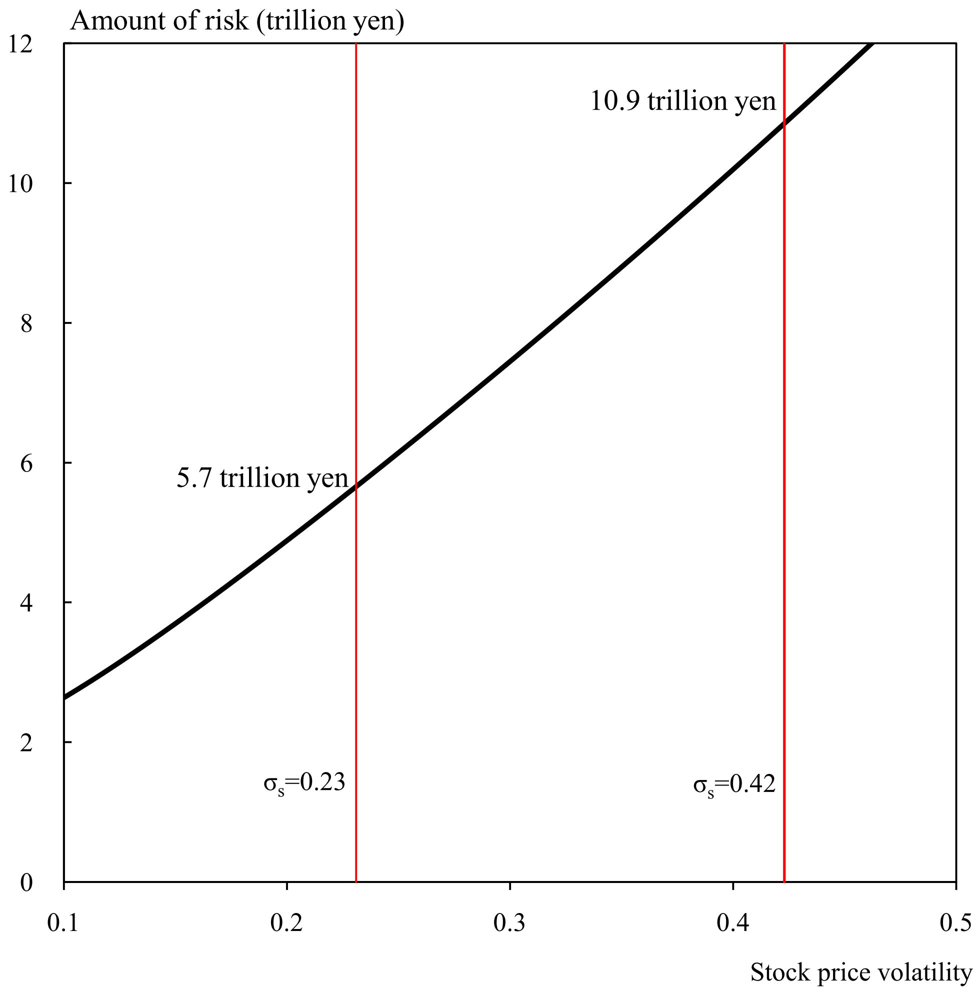
Changes in Correlation Coefficients and Amount of Risk (Regional Banks)



Note: The vertical lines, from left to right, represent the average level since 2000 (= 0.33), no correlation (= 0) and the realized value after the collapse of the bubble economy (= −0.63), respectively.

(Figure 9)

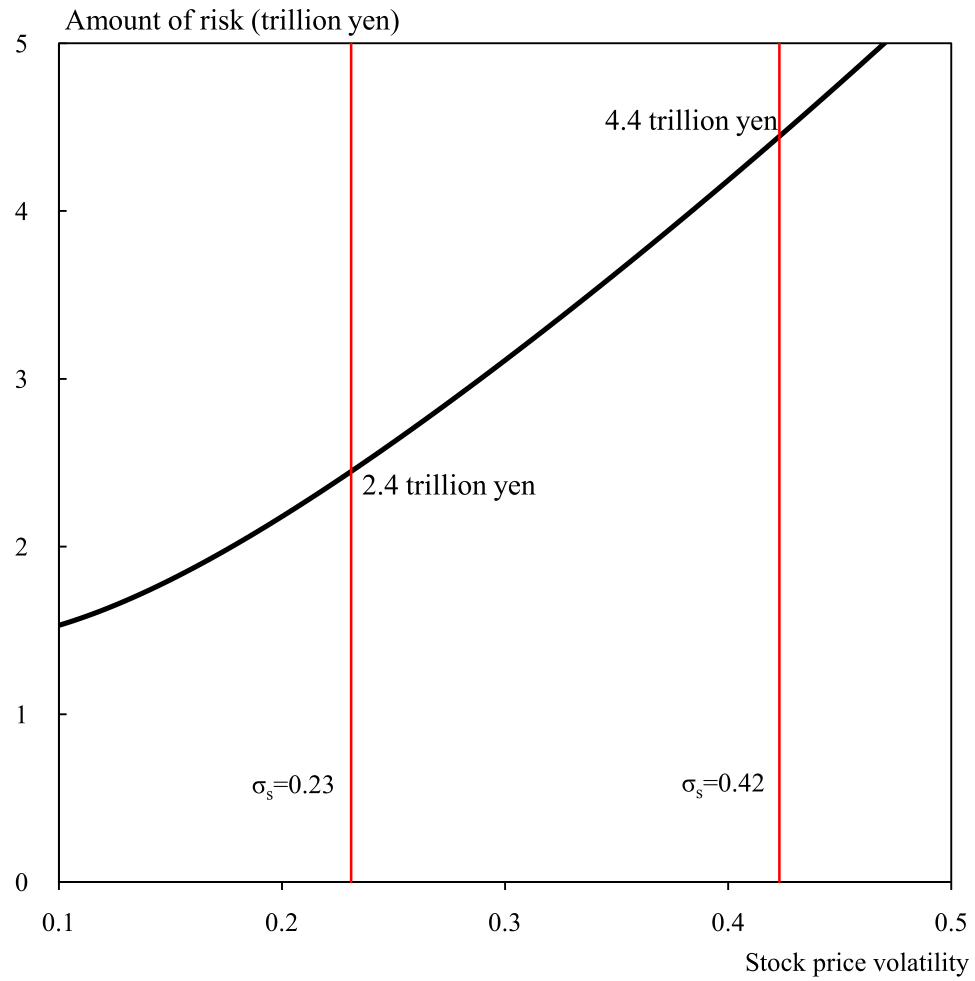
Changes in Stock Price Volatility and Amount of Risk (Major Banks)



Note: The vertical lines, from left to right, represent estimates based on the data since 2000 (= 0.23) and estimates based on the data for 1 year including the Lehman Shock (= 0.42), respectively.

(Figure 10)

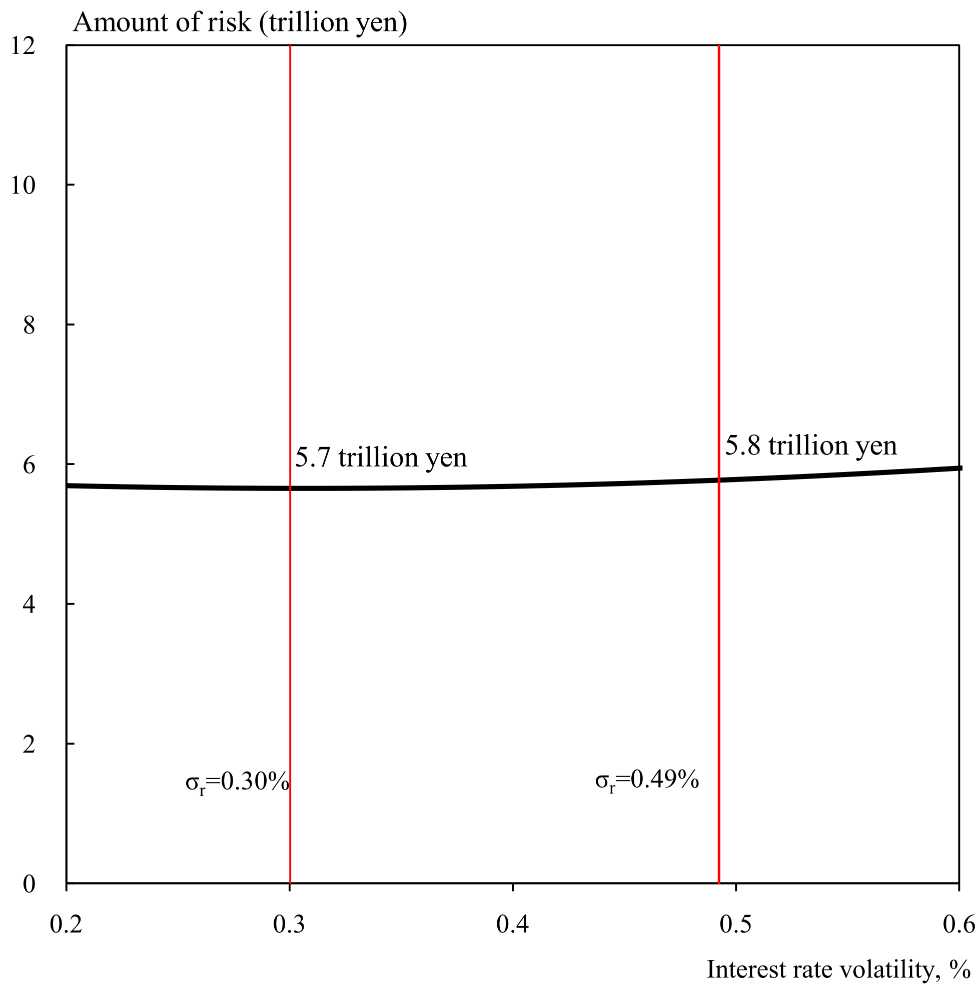
Changes in Stock Price Volatility and Amount of Risk (Regional Banks)



Note: The vertical lines, from left to right, represent estimates based on the data since 2000 (= 0.23) and estimates based on the data for 1 year including the Lehman Shock (= 0.42), respectively.

(Figure 11)

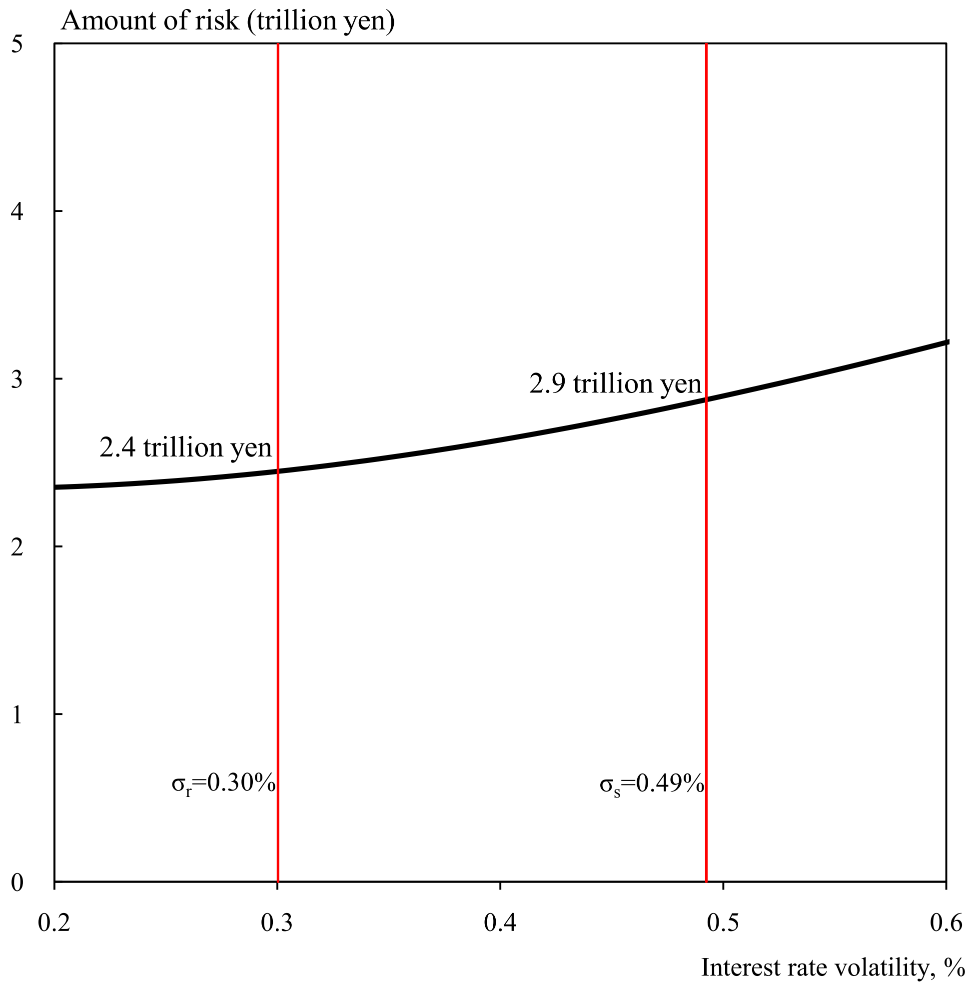
Changes in Interest Rate Volatility and Amount of Risk (Major Banks)



Note: The vertical lines, from left to right, represent estimates based on the data since 2000 (= 0.30%) and estimates based on the data for 1 year including the Lehman Shock (= 0.49%), respectively.

(Figure 12)

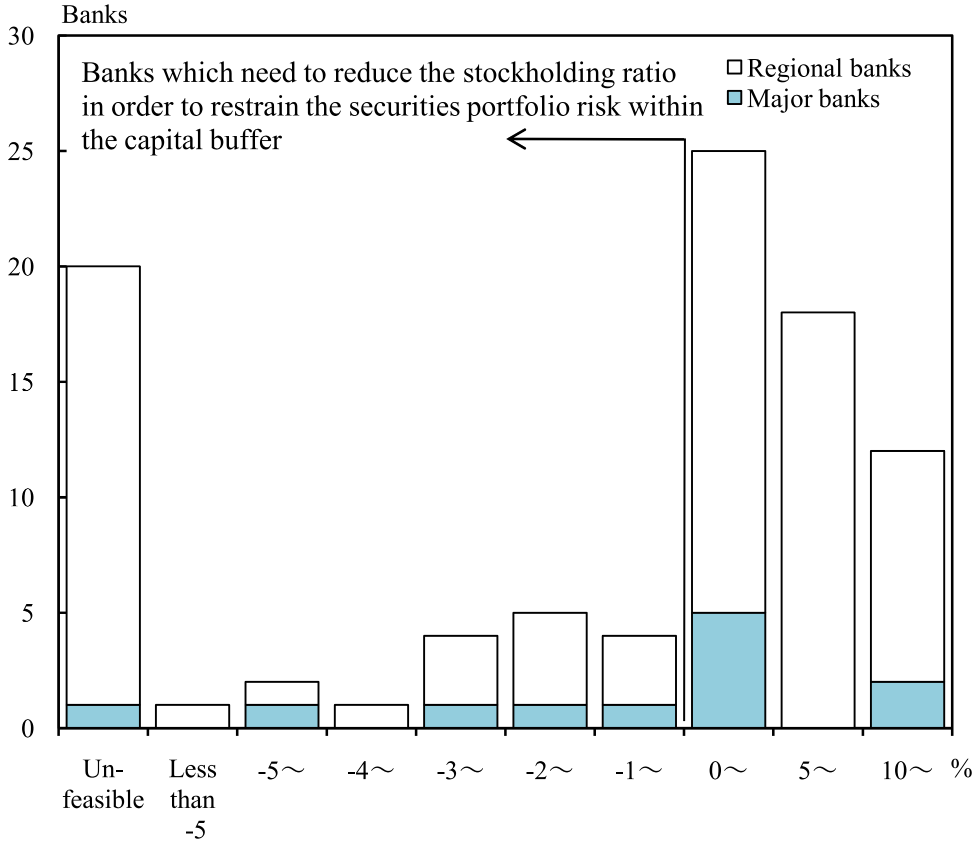
Changes in Interest Rate Volatility and Amount of Risk (Regional Banks)



Note: The vertical lines, from left to right, represent estimates based on the data since 2000 (= 0.30%) and estimates based on the data for 1 year including the Lehman Shock (= 0.49%), respectively.

(Figure 13)

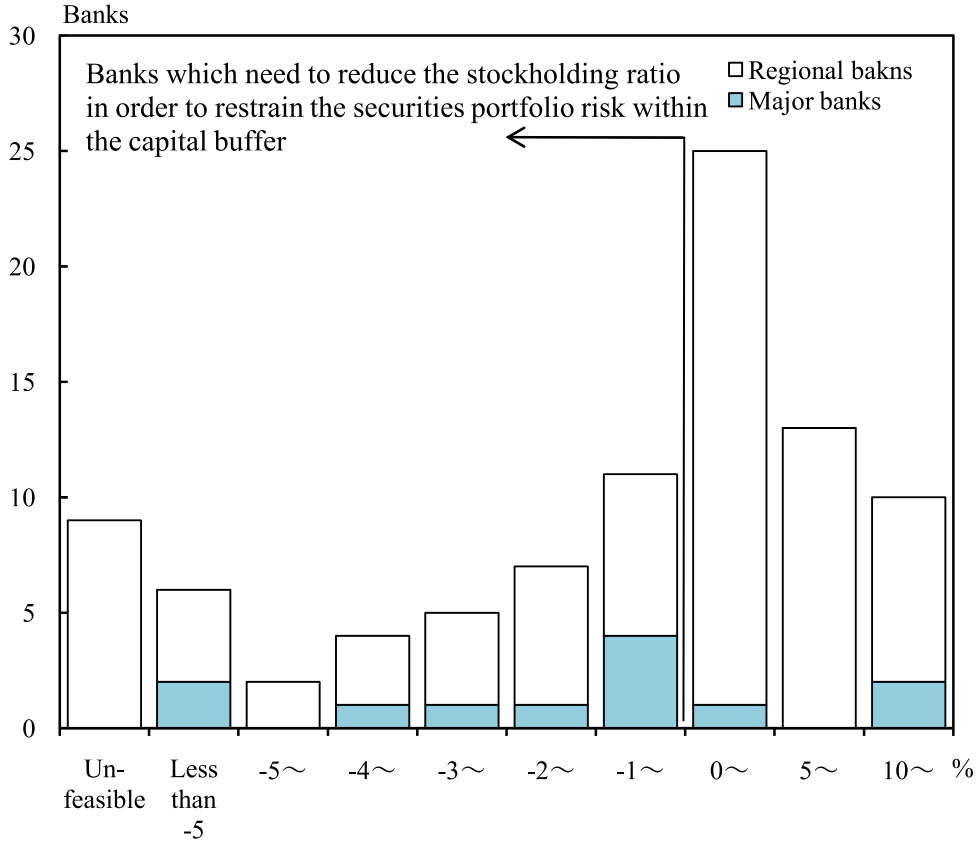
Comparison of Optimal Stockholding Ratio and Current Level of Individual Banks (In a Normal Market Environment)



Note: The banks which hold a positive capital buffer are listed. “Unfeasible” refers to a bank which cannot restrain the securities portfolio risk within the capital buffer despite any change in the stockholding ratio.

(Figure 14)

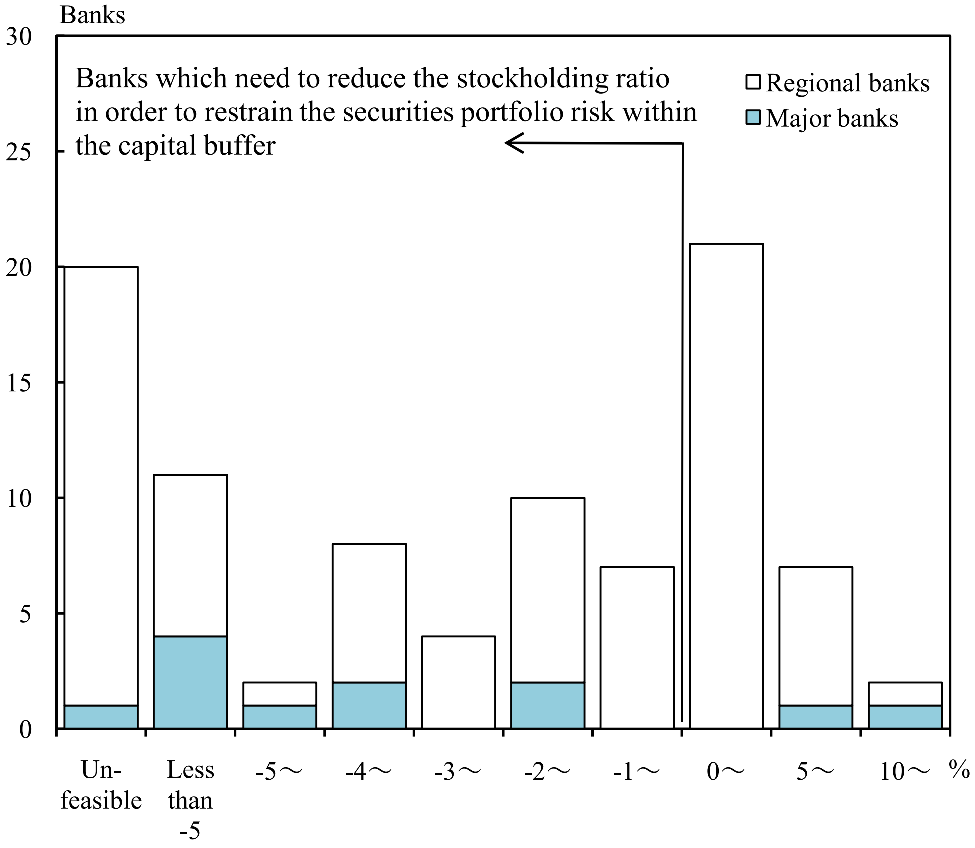
Comparison of Optimal Stockholding Ratio and Current Level of Individual Banks (Reversal of Correlation)



Note: The banks which hold a positive capital buffer are listed. “Unfeasible” refers to a bank which cannot restrain the securities portfolio risk within the capital buffer despite any change in the stockholding ratio.

(Figure 15)

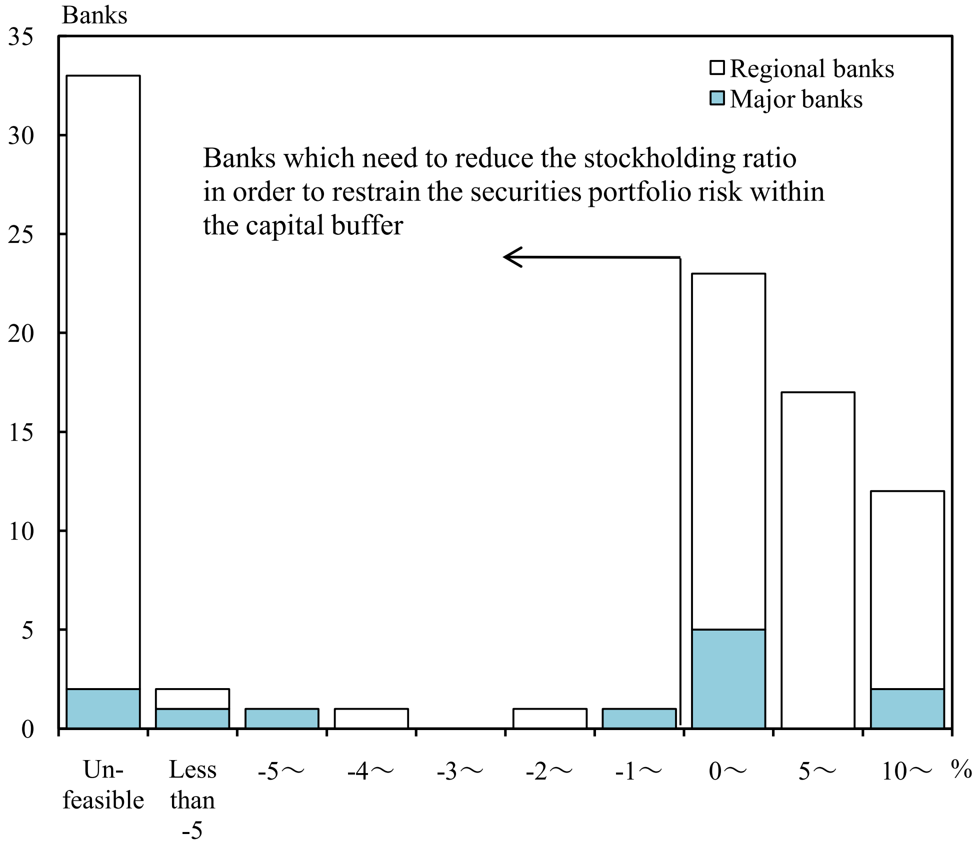
Comparison of Optimal Stockholding Ratio and Current Level of Individual Banks (a Rise of Stock Price Volatility)



Note: The banks which hold a positive capital buffer are listed. "Unfeasible" refers to a bank which cannot restrain the securities portfolio risk within the capital buffer despite any change in the stockholding ratio.

(Figure 16)

Comparison of Optimal Stockholding Ratio and Current Level of Individual Banks (a Rise of Interest Rate Volatility)



Note: The banks which hold a positive capital buffer are listed. "Unfeasible" refers to a bank which cannot restrain the securities portfolio risk within the capital buffer despite any change in the stockholding ratio.

(Table 1)

Basic Statistics

	Mean	Median	Maximum	Minimum	Standard Deviation	Skewness	Kurtosis	Number of Observation
Stock price	1,366	1,270	2,125	847	340	0.59	-0.91	2,922
Stock return	-0.02	0.02	12.86	-10.01	1.46	-0.34	6.26	2,922
Interest rate	0.45	0.31	1.24	0.08	0.29	0.76	-0.78	2,945
Change in interest rates	0.00	0.00	0.14	-0.12	0.02	0.35	4.99	2,945

Note: The stock return and the interest rate in unit of % and the change in interest rates in percentage points. The stock return excludes dividends.

(Table 2)

Values of Estimated Parameters

Parameter	Benchmark	In times of stress
μ	7.77%	—
σ_s	23.1%	42.4%
θ	0.45%	—
σ_r	0.30%	0.49%
κ	0.52	—
ρ	0.33	-0.63