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Reviewing US Monetary Policy in Disinflation Era: A Primer

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Reviewing US Monetary Policy in Disinflation Era: A Primer[*]

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Abstract

This paper reviews the experience of US monetary policy from 2000 to shed some light on issues regarding the effectiveness of monetary policy in a low inflation era. Our analysis is twofold. First, based on a simple inflation forecast targeting model introduced in Svensson (1997) and Kato and Nishiyama (2002) as its variant, we demonstrate that the actual federal funds rate closely followed its optimal path predicted by the model from the late 90s to mid-2003. Second, we examine the response of long-term interest rates or an implied forward curve to FOMC's policy changes by employing a version of the new IS/LM model. The result shows that the observed financial market response to the FOMC's statement released in August 2003 can be no less consistent with an effective change in the expectation of the degree of interest rate inertia than a failed policy commitment. Our simulation suggests that we cannot conclude that the Fed's commitment was ineffective during the recent phase of stagnation.

JEL Classification: E5, E4, C6

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1 Introduction

This paper reviews US monetary policy, especially focusing on the period from 2000 to mid-2003. In the period, the Federal Reserve Board (denoted as the Fed hereafter) cut the federal funds rate by 550 basis point down to a level of 1.00 percent in 2003. This is the lowest level during the last four decades. Obviously, this sequence of drastic interest rate cuts was a policy response to the downturn of the US economy in 2001, accelerated by the increasing threat of deflation or the risk of being caught in a liquidity trap. The Fed has recognized the potential harm of deflation as a "current and present danger" since the early phase of the recent recession. Ahearne et al. (2002) empirically investigate the Japanese experience in the 90s and concludes that the Bank of Japan's monetary policy easing was ex-ante adequate, but proved to be too tight ex-post, in that the Bank of Japan did not take out sufficient insurance against a downside risk, a liquidity trap. As the title of the paper by Ahearne et al. indicates,[1] they were well-aware of the new danger created by deflation and were trying to learn lessons from the Japanese experience in the 90s on how to prevent such a disaster through effective monetary policy reactions. Our study is partially motivated by Ahearne et al's (2002) paper. Namely, after its release, the US inflation has continued to fall to its current level of around 1.5 percent and thus the risk of falling into a liquidity trap has remarkably increased. The natural question which this paper addresses is whether the Fed has been delivering better policy to prevent deflation under the present low inflation predicament. Our answer to this question is, essentially, yes. Our main findings suggest that it is not easy to reject the hypothesis that the Fed has achieved a good performance so far in terms of multiple evaluations.

In this paper, we apply two approaches, each depending on slightly different models, to examine the effectiveness of monetary policy conducted by the Fed after 2000. First, we directly evaluate actual federal funds rate level in terms of an optimal monetary policy perspective assuming the actual state of the economy in each period as given. The preceding studies show the actual federal funds rate levels in recent years are significantly lower than those implied by the naive Taylor rule. However, we found that this comparison does not make much sense, once we take into account the potential risk of falling into a liquidity trap created by the zero lower bound on nominal interest rates. We showed that any simple linear rule, Taylor rule for instance, cannot be optimal in the presence of a zero lower bound on nominal interest rates, since the risk of falling into liquidity trap is a highly non-linear function of distribution of shocks to the economy

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[1] See "Preventing Deflation: Lessons from Japan," by Aheane et al (2002).

as well as the state of the economy. The effects of zero lower bounds have been viewed as a potential risk by many central bankers inside and out of the US in the recent low inflation era. For instance, Blinder (2000) warns succinctly “Don’t go there.” This is one of the clearest statements describing how the risk of the zero lower bound would easily make monetary policy less effective in stimulating the economy. This type of idea, which tries to prevent a non-negativity constraint, is recognized as “preemptive” monetary policy. The idea is first advocated informally by Goodfriend (1993, 2000)[2] and later thoroughly investigated by Orphanides and Wieland (2001) and Kato and Nishiyama (2002). Preemptive monetary policy can be easily understood with an analogy of liquidity constraints on consumers. When liquidity constraints are incorporated, a consumer is willing to increase her savings to avoid risk, since she cannot borrow as much as she would need in the absence of such constraints. Similarly, in the presence of a zero lower bound on nominal interest rates, there should be a very good reason for a central bank to be preemptive. Namely, its monetary policy should be more expansionary than it would be without such a constraint. In this sense, there is no rationale for a central bank to “save ammunition” when there is the risk of falling into a liquidity trap. Rather, they should use their ammunition more quickly and aggressively. This is the very idea of preemptive monetary policy as discussed in earlier studies. One of our motivations to conduct this research is to review the Fed’s actual monetary policy after 2000 in terms of these preemptive monetary policy perspectives. An earlier study along the same lines is Ahearne et al (2002), but their main focus was not on the US but the Japanese experience in the 90s, although their ultimate interest was in the US economy. Their punch line was straightforward; the Bank of Japan’s policy reactions were not that far from being optimal ex-ante, but ex-post inefficient, not preemptive enough, when explicitly considering the existence of the zero lower bound on nominal interest rates. In this paper, we examine the degree of such preemptiveness of the Fed policy reactions to find that since 2000, the Fed policy reaction has been very closely followed the optimal levels of the federal funds rate predicted by a formal optimality conditions.

Basically, our first approach employs a mechanical instrument rule and compare theoretically-implied levels with those of the actual federal funds rate. To set this simple instrument rule approach as a benchmark has a certain advantage over other more complex strategies. We know that mechanical instrument rules are not what the Fed’s decision process employs. Nonetheless, this approach carries some insight as it enables

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[2] Preemptiveness in the sense of avoiding deflation is referred to in Goodfriend (2000), while Goodfriend (1993) mentioned it in discussing inflation.

us to identify the judgemental portion of the policy reaction. Despite its simplicity[3] of our approach, it reveals actual federal funds rate data to be fairly consistent with those predicted by the optimal monetary policy, especially after 2001. The upshot of the first approach is simply that the degree of Fed's preemptiveness after 2001, which appears to be substantially discretionary as measured by the Taylor rule, has been close to optimal from the viewpoint of optimal policy rules that takes into account a zero lower bound on nominal interest rates.

As the first half of this paper demonstrates, policy review based on a mechanical instrument rule is informative in obtaining an illustrative evaluation. In the real-world, however, instrument rules are nothing more than guidelines and a great deal of actual monetary policy is conducted based on judgemental decisions. This judgemental reaction can be well supported by previous theoretical studies such as Currie and Levine (1993), Woodford (1999), and Svensson and Woodford (2001). In their models, optimal monetary policy is not a simple mechanical instrument rule, but depends on the entire history of policy reactions. Hence, the optimal policy reaction is time-variant and a complex function of an infinite number of arguments. Obviously such a complex function cannot be a prescribed guideline for monetary policy, since it is neither accountable nor transparent. The major difference between the first approach and the Currie-Levine-Woodford models lies in the treatment of the private sector's expectation. One implicit assumption behind the first approach is to omit an expectation channel of monetary policy transmission. However, once we incorporate the non-trivial role of the private sector expectations, no simple and mechanical rules can be optimal in stabilizing the economy as just demonstrated in Currie and Levine's study. This is a major reason why judgemental policy action is prevailing among many central banks in the real world. The discretionary portion of policy making could, if successfully implemented, be an effective suboptimal reaction that can replace the rigorous optimal reaction implied by Currie-Levine-Woodford framework.

In the second approach, we focus on the role of the expectation channel in monetary policy. The forward-looking nature of intertemporaly optimizing private agents shows it essential to analyze the role of expectations in investigating the efficacy of monetary policy. More specifically, here we examine the actual response of implied forward rate

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[3] The policy rule suggested in our paper is not "simple" in that it takes the form of a highly nonlinear function of the current state of the economy. "Simplicity" here means the approach is a simple comparison of actual data and theoretical value predicted by a mechanical policy reaction function, whatever complex function it appears to be.

curve to various policy actions of the Fed. In August 2003, the Fed announced that they would maintain accommodative policy stances "for a considerable period." This can be regarded as a kind of commitment for future policy actions. A typical view regarding the result of this commitment is that the Fed failed to exploit the intended effects of such commitment, since soon after the announcement, the slope of the implied forward rate curve on treasury bonds steepened rather than flattened, with a rise in long-term interest rates.[4] Such a view implies that the commitment was not credible, and thus the private sector (market participants) anticipated a tighter monetary policy in the future, despite the Fed's commitment to maintain an expansionary policy for a considerable period.

In contrast, we present a possible counter-argument to this view. We propose an alternative interpretation for the observed change in the implied forward rate curve. Suppose that the commitment to maintain a low interest rate policy is fully credible. Then, lower nominal/real interest rates would stimulate the economy and thus both output and inflation would rise in the near future. Provided that part of the Fed's policy reaction is based on a Taylor type endogenous feedback rule, such a boost to the economy will induce higher nominal interest rates, perhaps eventually, via this endogenous portion of the Fed's policy reaction. It should be noted that this argument could result in an opposite outcome from views stated in earlier studies, saying that a credible commitment to maintain a *low* interest rate policy leads to *lower* interest rates. Instead, our counter-argument can open the way for a possibility that a credible commitment to *low* interest rates would result in *higher* (long-term) nominal interest rates and stronger economic activity in the following period. Essentially, this seemingly contradicting arguments can be reconciled by a simple exercise of identifying the endogenous fluctuation of economic variables. The response of nominal interest rates reflects a mixed outcome induced by both exogenous policy actions, such as structural policy changes, and an endogenous reaction to fluctuations in the economy. Therefore, it is possible that if an initial interest rate cut is purely exogenous, the resulting response of nominal interest rates will not necessarily be a decline, since the latter effect of endogenously anticipated reactions might dominate the initial impact of the exogenous policy action. In this case, expected gradual rises in the nominal interest rate, in other words, a steepened implied forward rate curve, may be a reasonable outcome from such a policy commitment.

This paper uses a standard New-Keynsian dynamic general equilibrium model to

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[4] Okina and Shiratsuka (2003) report the empirical evidence that lower long-term interest rates were observed when the Bank of Japan announced a commitment to zero interest rate policy in 2002, although they provided neither identification nor interpretation for the observation.

simulate the typical response of long-term nominal interest rates or implied forward rate curves to a combined shock, consisting of an adverse IS shock and changes in policy reaction structure including the varying degree of commitment in particular. We will demonstrate that, given the estimated parameters of the US economy, a commitment to maintain low interest rates for a sufficiently long period can give a boost to the economy. This will give rise to a steeper implied forward rate curve than the curve under a commitment to accommodative policy stances for a shorter period. Hence, we cannot conclude that the Fed's commitment was ineffective solely because of the observed steepened implied forward rate curve following FOMC's commitment to longer-lasting low interest rates policy.

Before getting into our analysis, we mention one caveat to our strategy. Namely, in the first approach, we explicitly analyze the role of non-negativity constraints on nominal interest rates, but omit the expectation channel of monetary policy. On the other hand, the second approach does not involve the potential influence of the zero lower bound in particular on expectations. In short, we do not combine everything in the unified model, but we analyze each factor separately using simple models. This is because it is highly uncertain how a zero lower bound on nominal interest rates affects the private sector's formulation of its expectations. Although potentially helpful models are available, we do not know much about a sufficiently reliable theory on the relation of zero lower bound and the private sector expectations.[5] Moreover, those models, especially imperfect information models are highly abstract and not tractable enough for a practical policy analysis at this moment. In light of our purposes, an empirical and practical policy analysis of the actual US monetary policy, we find that our "hybrid" approach is a good primer providing a competent framework despite its naive nature.

The rest of this paper is organized as follows. Section 2 presents an illustrative evaluation of the federal funds rate level within the framework of simple inflation forecast targeting introduced in Svensson (1997) and its modified version, Kato and Nishiyama (2001). Section 3 incorporates the role of an expectation channel of monetary policy,

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[5] Promising models have been developped in Benhahib et al (1998, 2000), Eggertsson and Woodford (2003), and Evans and Honkapohja (2003) among others. However, we are aware that such models are potentially flawed for the following reason. Sticky *price* models (or the new Keynsian Phillips curve) are not supported empirically by actual US data. Instead, actual US data seems consistent with sticky *inflation* models, which were recently introduced in Mankiw and Reis (2002) and Woodford (2002). In many cases, to generate sticky inflation in a dynamic general equilibrium model requires some kinds of asymmetric/imperfect information. As mentioned above, such theories regarding information structure and zero lower bounds are now awaited in this literature.

which is not explicitly considered in the section 2 framework, into the model and examines the response of implied forward rate curves. After discussing some empirical issues regarding the recent US monetary policy, section 4 will conclude the paper.

2 Evaluating the level of federal funds rates in the 2000s

There is a broad consensus that the Fed controls the federal funds rate as its monetary policy instrument. Our first approach here is to evaluate its level against theoretical ones predicted by an optimal monetary policy reaction function. The comparison shows the extent to which it follows a simple reaction function and thus deviates from it. Our analysis is conducted in a similar fashion to Ahearne et al (2002), but differs in terms of the treatment of the zero lower bound on nominal interest rates. Due to the non-negativity constraints, we allow non-linear reaction functions instead of naive linear rules as shown in Ahearne et al (2002)[6]. In the following subsection, we will introduce the framework to derive optimal non-linear reaction functions.

2.1 A simple model for optimal monetary policy

Here we introduce the standard framework to analyze optimal monetary policy and its modifications when the zero lower bound on nominal interest rates is incorporated. As demonstrated in Ball (1997) and Svensson (1997), the linear policy reaction function, known as the Taylor rule first presented in the seminal paper, Taylor (1995), can be a solution to a dynamic optimization problem of a central bank in minimizing variance in inflation and output gap. Let π and y denote the inflation rate and output gap, respectively, and let denote π^* the target inflation rate of the central bank. Suppose that the central bank's period-by-period loss function is written as

$$L_t = \frac{1}{2}\left\{y_t^2 + \lambda\left(\pi_t - \pi^*\right)^2\right\}, \tag{1}$$

where λ is a positive weight that represents the central bank's preference. Following Ball(1997) and Svensson (1997), we assume a backward-looking economy consisting of conventional IS-AS formulation such that

$$\begin{aligned} y_{t+1} &= \rho y_t - \delta\left(i_t - E_t\pi_{t+1}\right) + \varepsilon_{t+1} & (2)\\ \pi_{t+1} &= \pi_t + \alpha y_t + \zeta_{t+1}. & (3)\end{aligned}$$

Equation (2) and equation (3) stand for aggregate demand and supply function where ε and ζ are random disturbances, which we call IS and AS shock respectively. Although

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[6] The role of non-linear reaction function is also mentioned in the appendix of Ahearnes' paper.

equation (2) includes a forward-looking variable, this can be substituted out through $E_t\pi_{t+1} = \pi_t + \alpha y_t$. The central bank's problem is now formulated as an intertemporal minimization problem with the objective,

$$\min : E_t \sum_{i=0}^{\infty} \beta^i L_{t+i} \tag{4}$$

subject to $y_{t+1} = (\rho + \alpha\delta) y_t - \delta\left(i_t - \pi_t\right) + \varepsilon_{t+1}$ and equation (3). Here, we add one more constraint to the model accounting for the risk of falling into liquidity trap. To do so, we explicitly state the non-negativity constraint on nominal interest rate, such that

$$i_t \geq 0,$$

and apply the Kuhn-Tucker conditions following Watanabe et al. (2001). Kato and Nishiyama (2002) consider the resulting effect of this additional constraint in detail. The upshot of their analysis is the modified optimal reaction function as follows:

$$i_t^* = \pi_t + \left(\alpha + \frac{\rho\theta_1 + \theta_1 - 1}{\delta\theta_1}\right) y_t + \left(\frac{\theta_1 - 1}{\alpha\delta\theta_1}\right)(\pi_t - \pi^*) + \underbrace{\left(\frac{1}{\delta\theta_1}\right)\sum_{i=0}^{\infty}\theta_2^i E_t \Psi_{t+i}}_{\Theta}, \tag{5}$$

where,

$$\begin{aligned}
\Psi_t &= \left(\rho\beta\psi_{t+2} - (1 + \rho + \alpha\delta)\psi_{t+1} + \beta^{-1}\psi_t\right)\delta^{-1} \\
\theta_1 &= \frac{\alpha^2\beta\lambda + \beta + 1 + \sqrt{(\alpha^2\beta\lambda + \beta + 1)^2 - 4\beta}}{2}, \quad \theta_1 \geq 1 \\
\theta_2 &= \frac{\alpha^2\beta\lambda + \beta + 1 - \sqrt{(\alpha^2\beta\lambda + \beta + 1)^2 - 4\beta}}{2}, \quad 0 < \theta_2 < 1.
\end{aligned} \tag{6}$$

ψ_t represents the Lagrange multiplier for the non-negativity constraint on nominal interest rates. Note that the optimal reaction function eqn(5) is linear in state variables π_t and y_t except for the fourth term, Θ. As Θ is a function of $E_t\psi_{t+i}$, there is no reason to believe that $\psi\left(y_t, \pi_t\right)$ is linear in its arguments. Suppose there is no zero lower bound so that $\Psi_{t+i} = 0$ for any i. Then the optimal reaction is linear and mirrors the familiar result from Svensson (1997a), which can be regarded as a 'Taylor class' rule. Once the zero lower bound is introduced however, even when this constraint is not binding for the current period, there exists a non-zero probability that such constraint may be binding in the future, which alters the optimal monetary policy reaction from the case without the zero lower bound. Indeed this very factor is captured by Θ of equation (5).

The original Ball-Svensson set-up enables us to derive guidelines for an optimal monetary policy without a zero lower bound on nominal interest rates. Similarly, equation (5), the modified Taylor rule, would also provide implications about what is expected of a central bank when faced with the risk of falling into a liquidity trap. The guidelines for optimal monetary policy in the presence of the risk of liquidity traps are characterized by the forth term Θ in equation (5). Although an analytically explicit form of the term Θ does not exist, both qualitative and quantitative properties of Θ are examined thoroughly in Kato and Nishiyama (2001) and Kato (2003). Based on their investigation into Θ, here we have come to a proposition regarding optimal monetary policy under the risk of falling into a liquidity trap as follows.

Proposition 1 (Optimal monetary policy with a zero bound) *Let* $i^{Taylor}(\pi_t, y_t)$ *be the optimal monetary policy reaction function when there is no zero bound on nominal interest rates. Let* $i^*(\pi_t, y_t)$ *be the optimal monetary policy reaction function in the presence of a zero bound on nominal interest rates. Then for any state* (π_t, y_t) *where* i^* *is strictly greater than zero, the following inequalities are true.*

$$i^* \leq i^{Taylor} \tag{7}$$

$$\frac{\partial i^*}{\partial \pi_t} \geq \frac{\partial i^{Taylor}}{\partial \pi_t}, \frac{\partial i^*}{\partial y_t} \geq \frac{\partial i^{Taylor}}{\partial y_t} \tag{8}$$

$$\frac{\partial^2 i^*}{\partial \pi_t^2} \leq 0, \frac{\partial^2 i^*}{\partial y_t^2} \leq 0 \tag{9}$$

Proof. See Kato and Nishiyama (2001) and Kato (2003). ■

The inequalities presented above have the following interpretations. The first one, $i^* \leq i^{Taylor}$, implies the conventional wisdom described as "preemptive" monetary policy via interest rate control implementation. Namely, it states that monetary policy under the risk of falling into a liquidity trap will be more expansionary than what would be ideal without it, i.e., the standard Taylor rule. Inequality (8) provides qualitative instructions on how quickly a central bank must cut nominal interest rates in the face of a downturn of the economy. In other words, inequality (8) states that a central bank should be more aggressive than the Taylor rule would suggest. For the last condition, inequality (9) predicts that for any discrete change in the state of the economy, a central bank's expansionary reaction should be larger than its contractionary one. Combining inequalities (7), (8) and (9), the essential idea is that optimal monetary policy is monotonically increasing and concave in inflation and output gap, and has a steeper slope than the Taylor rule. A straight interpretation of the proposition is that a central bank is required to take a bold action when the economy is on the brink of deflationary spiral. Although many central bank economists have informally referred to similar suggestions regarding the qualitative nature of monetary policy in such a stress as discussed here, neither rigorous proof nor precise inspection of the optimal rule with the zero bound has been provided until recently. This is because the optimal rule, denoted as equation (5) cannot be written out in an explicit form. Hence it requires a numerical simulation technique to approximate the shape of the function, which is indispensable to present an illustrative comparison of actual federal funds rates with those predicted by the model. In this paper, we apply a unique technique called the Collocation method to precisely compute the approximate shape of the reaction function.[7] In the following subsection, we will evaluate the actual level of federal funds rate through the numerically approximated optimal rule using the Collocation method.

2.2 Numerical results: Did they save the ammunition?

Before evaluating the actual federal funds rates within the framework introduced in the previous section, we present a simple comparison of the actual federal funds rates with the standard Taylor rule. We will use a 1.5 coefficient for inflation and 0.5 for output gap as first introduced in the seminal paper by Taylor (1983). Figure 1 illustrates this comparison. It reveals that the actual federal funds rates (denoted by solid line) declined faster than would be implied by the standard Taylor rule (thinner line). For instance, as of the beginning of 2003, the standard Taylor rule requires the federal funds rate to be at about 3 percent whereas the actual federal funds target stood at 1.25 percent. This fact directly suggests that the recent policy stance of the Fed has been more preemptive than implied by the Taylor rule.

Now our first question to be addressed here in this paper is whether Fed's policy has really been preemptive enough under the risk of "unwelcome fall in inflation." It turns out that the comparison of actual rates and the Taylor rule does not make much sense, once we consider the potential risk of falling into a liquidity trap created by the zero lower bound on nominal interest rates. As discussed in the previous section, we have gauged the Fed's monetary policy from 1990 to mid-2003 in terms of the optimal monetary policy rule considering the existence of the zero lower bound on nominal interest rates.

Figure 1 compares the actual federal funds rate with the optimal rate predicted by

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[7] All parameter values are estimated by multiple GMM using actual US data for the last two decades. Estimation results are presented in table 2 of appendix C. For Fed's implicit target inflation rate, we set 1.5%. We will discuss this figure later in the following section.

the model[8] (dashed line). As shown in the figure, the two rates diverge sharply during the late 90s. The figure indicates that actual Fed policy was too tight over the period, although the Fed cut the federal funds rate in 1995 and in 1998. The Fed's policy stance in this period was somewhat puzzling in light of the corresponding levels of CPI inflation and output gaps, since CPI inflation growth was declining in the late 90s and output gaps remained negative until mid-1998 as indicated in figure 2. We do not know the exact reasons why the Fed kept the tighter policy stance at that time than the optimal rule. A popular view to this puzzle is that the Fed may have been concerned that lowering the federal funds rate might boost the bubble in asset prices, since the US stock prices were soaring during the late 90s.[9]

From the beginning of 2001 to mid-2003, however, the actual federal funds rate considerably agrees with those predicted by the optimal rule. In retrospect, the US stock market, which recorded its all-time high in March 2000,[10] plunged in the second half of 2000, while the growth rate of industrial production was slowing. In response to those signs of downturn of the economy, the Fed decided to cut the federal funds target rate at the beginning of 2001. This reduction was the first of a series rate cuts that brought the federal funds rate down by 550 basis points to a level of 1.00 percent, the lowest in about four decades. On the other hand, the figure 1 indicates that the optimal rate started to decline from the second quarter of 2000, which tells us that the actual federal funds rate lagged three quarters behind the optimal rates over a couple of years during the corresponding period. It might not go unfair to draw the conclusion at this moment, however, that the illustrative evaluation seem to suggest that the Fed's monetary policy has been noticeably preemptive in the recent low inflation environment.

Here let us refer to the controversy regarding the aggressiveness of monetary policy in a low interest rate environment. Some market economists/practitioners say, mostly in casual talks and informal papers, that the Fed prepares for the worst-case scenarios and thus saves some possible actions for a situation where the federal funds rate is close to zero.[11] Obviously this view is opposed to the notion of preemptive monetary policy

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[8] We set 1.5% for Fed's target inflation rate in computing the optimal level of the federal funds rate as right in the middle of the 1-2 percent range following governor Ben Bernanke's working definition of "price stability." Since the Fed has not formally announced its target inflation rate, this figure is naturally debatable. However, even if the direct comparison of *levels* between optimal and actual FF rates might be disputable due to the uncertainty stemming from Fed's target inflation rate, *changes* of the FF rate from recent peak in 2000 to the bottom should be less problematic and consistent with the model's prediction at given levels of the target inflation rate.

[9] Among many studies providing similar views on this issue, see Cecchetti (2002) for instance.

[10] As measured by S&P 500 and NASDAQ.

[11] Among a great deal of this kind of arguments, a typical view can be found in an International

as discussed in the previous section. On the contrary, figure 1 shows that the Fed has not been keeping its powder dry, but aggressively cut the federal funds rate to try to preempt severe deflation/contraction of the economy.

To summarize, (i) in comparison with the standard Taylor rule, the Fed's monetary policy from the beginning of 2001 to mid-2003 has been noticeably preemptive. Further, (ii) the Fed's actual monetary policy has been nearly consistent with the optimal rule taking into account the presence of the zero lower bound on nominal interest rates, which leads us to conclude that it has not been keeping their powder dry.

3 Incorporating the expectation channel: the role of commitment

In the second half of the paper, we will analyze the Fed's recent monetary policy from another perspective. Unlike the first half where we applied the framework of optimal inflation forecast targeting, to provide an overall illustrative evaluation of the level of federal funds rates, here we will present empirical case studies focusing on the specific dates on which the Fed's important monetary policy actions were taken. This section specifically emphasizes the role of the expectation channel of monetary policy and the efficacy of commitment via the expectational channel of monetary policy. Our study presented here is partially motivated by the following episode: on September 15, 2003, the Fed held a meeting to discuss effective communication between monetary authorities and the financial markets; an underlying concern of the Fed was addressed to the views expressed by some practitioners in relation to the increased long-term rates following the FOMC statement released on August 12, 2003, which supposedly signified the failure of the Fed's policy commitment. Basically, we are skeptical of such views and present a counter-argument to demonstrate that such a rise in long-term interest rates or the steepened implied forward rate curve can either be consistent with a failure or a success of the policy commitment. The judgement depends on various parameter values of the US macroeconomy, and given our estimated parameter values, it is more likely that the observed rise in long-term interest rates proved the efficacy of the policy commitment rather than a failed one. The following subsection will introduce our argument with a brief review on the standard framework of monetary policy analysis when private agents are forward-looking.

Newsweek article dated March 1 2004, "an open letter to chairman Greenspan" written by Stephen Roach. In the letter, Roach urges chairman Greenspan to immediately hike the FF rate from 1 to 3% like "reloading the cannon in war" so as not to run out of ammunition.

3.1 Some theoretical issues regarding commitment

The simple model in the previous section ignores the role of the private sector's expectations. The reason we ignore the expectation channel is simply because the original Taylor rule cannot be consistent with any optimal feedback rule, once we incorporate the role of expectations into the framework. Since our purpose in the previous section was to compare the linear Taylor rule and the optimal/nonlinear Taylor rule with the actual data, we had to make all those models hold a footing on the same basis. For a more general argument, however, this assumption of no role for expectations, is obviously too specific. In this section we will apply the second approach, namely examining the efficacy of the Fed's monetary policy by focusing on its expectation channel. For this purpose, we will modify the model by adding expectational terms with respect to output and inflation as shown below. Those two terms incorporated in the model represent behaviors of standard utility-maximizing households and monopolistically price-setting firms with some menu costs. The modified model is as follows,

$$\begin{aligned} y_t &= \rho y_{t-1} + (1-\rho) E_t y_{t+1} - \delta(i_t - E_t \pi_{t+1}) + \varepsilon_t \quad (10) \\ \pi_t &= \gamma \pi_{t-1} + (1-\gamma) E_t \pi_{t+1} + \alpha y_{t-1} + \zeta_t . \quad (11) \end{aligned}$$

As shown here, this model can be regarded as a variant of the new IS/LM model, (or "hybrid" IS and Phillips curve model) which is commonly used for monetary policy analysis in literatures. It is well known that these two equations can be derived from the first order conditions of rigorous dynamic optimization problems of forward-looking households and imperfectly competing price-setting firms, except for lag terms appearing in each equation.[12] We do not provide micro-foundation of them in this paper, but instead clarify the relationship between this model and the one introduced in the first half of this paper. Let r_t denote the real federal funds rate. Then, rewriting equation (10) leaves,

$$\begin{aligned} \delta r_t &= (1-\rho) E_t y_{t+1} - y_t + \rho y_{t-1} \\ E_t y_{t+1} &= \frac{1}{\tilde{\theta}_1} y_t - \frac{\delta}{\tilde{\theta}_1} E_t \sum_{j=0}^{\infty} \tilde{\theta}_2^j r_{t+1+j} \\ &\equiv \tilde{\rho} y_t - \tilde{\delta} r_t^L \quad (12) \end{aligned}$$

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[12] See Yun (1996) or Gali (2002) for the detailed derivation of these equations from the first-order conditions.

where $\tilde{\rho} \equiv 1/\tilde{\theta}_1$, $\tilde{\delta} \equiv \delta/\tilde{\theta}$, long-term real interest rate defined as $r_t^L \equiv E_t \sum_{j=0}^{\infty} \tilde{\theta}_2^j r_{t+1+j}$ and $\tilde{\theta}_1$ ($\tilde{\theta}_2$) is the larger (smaller) root of the characteristic equation, $(1-\rho)z^2 - z + \rho = 0$. Note that the redefinition $r_t^L \equiv E_t \sum_{j=0}^{\infty} \tilde{\theta}_2^j r_{t+1+j}$ indicates that the weighted average of the real federal funds rates can be interpreted as a quasi-long-term real interest rate.[13] With all those redefined parameters, the resulting IS equation looks quite similar to equation (2) shown in the previous section, with the real federal funds rate replaced by the long-term real interest rate. Hence, incorporating the forward-looking variable, $E_t y_{t+1}$ into the IS equation in the first model is nearly equivalent to replacing the real federal funds rate with the long-term interest rate.

Next, we will briefly discuss the optimal/sub-optimal monetary policy in these types of the new IS/LM model. Woodford (1999) demonstrated that the optimal monetary policy in this framework cannot be expressed only through state variables of the economy, but it also depends on the entire history of a central bank's policy actions. Intuitively, efficient monetary policy needs to exploit strong commitment effects by changing the private sector's behaviors via a commitment to certain future policy actions. In general, the optimal history-dependent policy reaction is not a simple reaction function, but rather complex conditions consisting of an infinite number of arguments. Since such a complex policy reaction function is not a realistic guideline for central banks to rigorously follow, in this paper we consider a suboptimal rule in the simple form as shown below.

$$i_t = \underbrace{\phi i_{t-1} + a y_t + b\left(\pi_t - \pi^*\right)}_{\text{endogenous feedback rule}} + \eta_t \tag{13}$$

This type of reaction function including interest rate inertia (captured by the ϕi_{t-1} term) is widely recognized as one of the most empirically plausible specifications that imitates the actual Fed's policy in recent decades. Theoretically, the AR(1) smoothing term in equation (13) plays a significant role in approximating the optimal history-dependent policy as discussed in Woodford (1999). The residual term η_t stands for the discretionary portion of the Fed's policy actions. This term η_t needs to be explained to avoid confusion. The policy rule fully represented in equation (13) is consistent with a policy commitment via interest rate inertia. The private sector recognizes the structure of the (suboptimal) policy rule as shown in equation (13), and treat it as given. However, at the same time, it is aware of the existence of the η_t term and a possibility of deviation from a mechanical/endogenous feedback rule as described in equation (13). Whenever the Fed deviates from a mechanical feedback rule, which is frequently observed on monthly

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[13] In numerical simulations presented later in this paper, we compute long-term rates to be consistent with the 3-year moving average of federal funds rates. Namely, $i_t^L = \sum_{j=0}^{12} i_{t+j}/12$ where i_t^L stands for the 3-year nominal interest rate.

basis, the discrepancy η_t is perceived as a *discretionary* policy element. Hence in this framework, both commitment and discretion coexist in the same model. This seems more realistic than a purely theoretical commitment/discretion policy, as a thorough solution to the dynamic game problem between monetary authorities and the private sector.

3.2 Was the Fed's commitment ineffective?

3.2.1 A theoretical benchmark

Before turning to empirical studies on the relationship between the FOMC statements and the response from financial markets, let us consider a simple benchmark case to clarify the essence of our analysis. Suppose the Fed cut the federal funds rate exogenously to accommodate a downturn of the economy induced by a negative IS shock. Here we say "exogenously" presuming that the Fed's policy action is not incurred mechanically via an endogenous feedback rule, i.e., the Taylor rule, but via a discretionary change in policy action represented by η_t in equation (13). It should be noted that with a very small ϕ, *ceteris paribus*, such an initial discretionary policy action does not last and soon after the initial impact, diminishes quickly. In contrast, with a large ϕ, especially when close to unity, such an exogenous policy action is expected to persist for a considerable period. That is, a larger ϕ represents a "stronger" Fed commitment[14] to maintain the current policy stance for a longer period.

The question is how the implied forward rate curve and long-term interest rates respond to the combined shock, adverse IS shock and discretionary federal funds rate cut (exogenous expansionary policy action), as discussed above. Do we observe significant differences between the two cases, one with a smaller ϕ (=0.4 assumed here) and the other with a larger ϕ (=0.8)? Figure 3 indicates the response of an output gap, implied forward rate curve[15] and long-term interest rates to the combined shock.[16] The figure might seem striking. The most notable feature is how long-term interest rates in the lowest panel remain substantially *lower* with a smaller ϕ, i.e., a *weaker* commitment to

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[14] Note that here we use stronger (weaker) commitment for representing higher (lower) degree of interest rate inertia. We do not measure the strength of commitment in terms of credibility, but only by the magnitude of policy inertia under fully credible environment for any degree of such inertia.

[15] The data shown in the panel is not the implied forward rate curve, which is computed using swap rates, but the future path of federal funds rates. Precisely, they are not equal in the sense that the future federal funds rates are not "implied" in any sense here. Note, however, that the future sequence of short-term rates is conceptually equivalent to implied forward rates, since perfect foresight is implicitly assumed in our analysis.

[16] Matlab codes used for the simulations here are downloadable at `http://aa4a.com/kato/`.

maintain a low interest rate policy, than with a larger ϕ, i.e., a *stronger* commitment. Further, the middle panel displays that the federal funds rates also stay longer at lower levels with a smaller ϕ than with a larger ϕ. The figure might appear to be counter-intuitive, because a stronger commitment to a low interest rate policy followed by lower interest rates seems more natural as well as intuitive. However, the figure shows the exact opposite. Why is that?

Recall that the federal funds rate consists of two different types of terms in equation (13), the endogenous feedback rule and exogenous/discretionary policy elements represented by η_t. As discussed in the previous subsection, in an environment where the private sector is forward-looking, the monetary policy rule which exhibits some inertia, represented by positive ϕ here, would perform more effectively in stabilizing the economy. This means that policy action under a certain positive ϕ, i.e., monetary policy with a commitment, would be more effective with a help from the expectation channel than it would with $\phi = 0$, when the central bank intends to stimulate the economy in response to a negative disturbance to the economy. Hence, as a result of effective monetary policy with a reasonable (suboptimal) degree of commitment, it is very likely to observe output gap/inflation increasing more quickly than it would with an insufficient degree of interest rate inertia. In that case, long-term interest rates would shift higher with a larger ϕ than with a smaller ϕ due to the successfully increased output gap/inflation Generally, we cannot uniquely determine how (future) nominal interest rates will react to a combined shock, since the response is determined as a net effect of the two factors in different directions. Nonetheless, according to figure 3, we now know that it is not necessarily true that higher long-term interest rates prove a failed policy commitment, using US data from the last two decades[17], with our estimates on parameters, US monetary policy with a stronger commitment (larger inertia denoted by $\phi = 0.8$ here) to maintain an accommodative stance would be associated with higher long-term rates than that with a weaker commitment due to the quicker recovery in economic activity.[18]

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[17] Estimation results are reported in table 2 in the appendix C.

[18] There exsit some preceding empirical studies trying to measure the efficacy of monetary policy by examining the response (curvature and shift directions) of a yield curve. Those econometric approaches are potentially flawed, since it is not easy to distinguish the signs of a successfull outcome of monetary policy from that of failure only by estimation results. For instance, is an estimated downward shift of the yield curve a good or bad sign for the efficacy of monetary policy? Considering the results of our analysis, now we know that the answer is ambiguous. To distinguish one from the other, we need dynamic general equilibrium analysis, since it provides us with illustration on what we woud observe when monetary policy is effective/ineffective. On the other hand, econometric analysis based on a partial equilibrium approach only gives us a measurement, but we never know the interpretation of such

Our argument introduced here in this section is nothing new. It is a simple example of the more general argument presented in Woodford (1999) that demonstrated an optimal degree of inertia in the interest rate feedback rule in stabilizing the economy. Hence, our example is a natural consequence of Woodford's argument that where there is a more effective degree of commitment, the implied forward rate curve tends to steepen in response to stimulative monetary policy actions, since it *is more effective*.

3.2.2 Empirical analysis and related simulation results

Having confirmed the validity of a theoretical benchmark case, let us turn to actual data from the US financial markets when the Fed took policy actions in 2003. We will focus on three specific dates on which FOMC statements substantially affected financial market expectations. We will pay particular attention to one date which might be associated with commitment effects on the private sector's behavior. For this purpose, we will examine changes in implied forward rates and euro dollar futures rates observed on each corresponding date.

First, the FOMC released a statement that referred to an "unwelcome substantial fall in inflation" after the May 6 meeting. This statement seemed to suggest that the FOMC would implement unconventional monetary policy actions such as aggressively purchasing longer-term Treasury bonds in the near future since the market perceived that the Fed would run out of room for a conventional rate cut after the federal funds rate was lowered down to 1.0 percent. This announcement caused a significant shift of the implied forward rate curve and euro dollar future curve downward. In particular, 1 to 3 year-ahead forward rates and 4 to 6 quarter-ahead euro dollar futures rates sharply plunged as indicated in figure 4-1. This observation is a basic example of how financial markets (i.e., private sector) realize newly arrived information regarding a future adverse shock to the economy or future policy actions by the Fed. We put data from this typical episode into our model. Figure 7 shows simulated dynamic responses of our model to newly arrived information of future adverse shocks to the economy. Casual observation comparing the actual response (figure 4) with the simulated response (figure 7) reveals that our model using estimated parameter values can replicate actual financial market responses quite well in this case. The outcome of the model, which possibly implies what happened in the real world, can be explained as follows: on arrival of the announcement, the private sector revises its consumption/investment plans as well as expectations for future inflation; since they anticipate future disinflation, it is optimal for

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measurement without a structual general equilibrium model.

them to smooth out their consumption holding off current expenses, and putting them off for a couple quarters; this immediately lowers the output gap, but it is accommodated by lowered short-term rates via a Taylor-type endogenous stabilization effect of monetary policy reactions. Reflecting all these factors, long-term rates fell significantly after the announcement. This is a natural consequence of the weakened economic activity associated with uniformly lowered short-term interest rates.

Second, June 25 presents another interesting episode. The FOMC cut the federal funds rate by 25 basis points to 1.0 percent and alluded to a firmer spending, markedly improved financial conditions and labor and product markets that were stabilizing. The financial markets responded somewhat oddly. Indeed, the statement entailed a significant effect on interest rates. Figure 5 indicates changes in the implied forward rate curve and euro dollar future rates on June 25. This shows that both curves shifted upward despite the expansionary policy action announced by the FOMC. This seemingly contradictory observation can be reconciled when considering initial conditions in the financial markets. At that time of the announcement, many financial market participants were said to have anticipated a 50 basis point interest rate cut rather than a 25 basis point cut as table 1 shows. Hence, given such an initial information set in the financial markets, the FOMC announcement exerted a contractionary shock on the market, wiping out expectations for unconventional policy actions.

Table 1: Federal funds futures rate

Delivery month	FF futures rate
June 03	1.180
July 03	0.870
Aug 03	0.835
Sept 03	0.820
Oct 03	0.810

Note: As of June 24, 2003.

Figure 8 reports the simulation results. In contrast to the previous example, we observe some discrepancy between the actual and simulated responses of the financial markets. That is, on impact, changes in the implied forward rate surge significantly, but they are followed by lower interest rates for later periods. The initial surges in short-term rates in figure 8 are consistent with actual observations shown in figure 5. However, the future path of implied forward rates and long-term rates do not replicate actual responses in the US financial markets. Regarding this anomaly, a reasonable hypothesis that potentially fills this gap is that there were market expectations for "unconventional monetary policy actions," fostered by chairman Greenspan's June 3 speech for IMC in Berlin in addition to the FOMC statement of May 6. In an economic model "unconventional actions" are regarded as a complete structural change in the current and historical policy reaction function, such as equation (13) in our model. This is an out-of-scope phenomenon that our model cannot handle, since in order to obtain reasonable simulation results, the conventional policy reaction function, equation (13), must be replaced by something representing an "unconventional" reaction. However, no one knows exactly what constitutes unconventional monetary policy actions. This model uncertainty problem goes beyond this paper's analysis and such an anomaly should be resolved through further researches that explicitly employ techniques to handle uncertainty and heterogeneous information.

Our simulation results only imply that the financial markets very likely anticipated the Fed taking unfamiliar actions in the near future, and such expectations diminished on impact of the FOMC statement released on June 25. The withdrawal of such expectations that our simulation could not replicate, if considered properly, might fill the gap between the actual response and the simulated response.

3.2.3 The FOMC of August 12, 2003

Let us turn to the August 12 FOMC meeting. This third episode from which our model can derive implications on monetary policy impacts is the Fed's allusion to the length of period during which it maintains the policy stance expressed in the FOMC statement released on August 12. In this statement, the FOMC declared that the Fed would maintain current accommodative policy "for a considerable period." Figure 6 shows the observed response of the financial market. As depicted in figure 6, market participants had expected lower short-term interest rates not to last for a long period, (although we do not know the exact length of the "considerable" period,) but to eventually rise after a couple of years. Meanwhile, on August 12, long-term interest rates stayed almost intact or even increased a little instead of falling, as shown in figure 11. A naive interpretation of this market reaction is that the commitment was not perceived credible and therefore, the implied forward rate curve *steepened* rather than flattened in spite of the FOMC's commitment to maintain a low interest rate policy.

One possible interpretation of the August 12 FOMC statement in the context of our dynamic general equilibrium model is simply an unanticipated rise in policy inertia ϕ, since a larger ϕ represents a more persistent policy reaction as mentioned in the previous subsection. While we do not exclude the possibility of a less than credible commitment, our simulation results depicted in figure 9 seem to support this interpretation. At a closer look, the changes in the implied forward rate curve and long-term rates in figure 9 appear fully consistent with the actual observations reported in figure 6. Recall that our simulation presumes rational expectations with perfect information. Hence, by construction, no less than credible action allowed in the virtual economy is simulated in our analysis. Figure 9 depicts a simulated path of related variables in response to a shock, such that the degree of commitment, denoted by ϕ in equation (13), suddenly revised upward (from about 0.4 to 0.8, for example). As discussed in the previous subsection, a larger ϕ is consistent with more persistent policy reactions maintaining the current policy stance. Hence, a similar interpretation can be applied to this case. The suddenly realized larger ϕ leaves the federal funds rates lower for a "considerable" period, which in turn stimulates the economic activity. Now the private sector anticipates a change in lower future interest rates, and is willing to immediately increase their spending due to an intertemporal substitution. The increased spending/economic activity will push up the federal funds rates gradually, which will in turn offset their initial decline. The resulting observation in the implied forward rate curve could either be an upward/downward shift, since the direction of the shift depends on the net balance of the two opposite effects. Given the estimated parameter values in our model, it is likely that the implied forward rate curve steepens in response to the upward revision in the degree of commitment. This is because stronger economic activity associated with a higher federal funds rate in the future period would dominate the other effect of initial decline in federal funds rates. Consequently, the observed response of the long-term rates was not a decline, but a substantial increase reflecting the steepened implied forward rate curve. In the model, the relationship of short- and long-term rate is defined as

$$i_t^L = \frac{1}{12}\sum_{j=0}^{12} i_{t+j}$$

where i_t^L denotes the 3-year nominal interest rate. Via this relation, the steepened implied forward rate curve can be consistent with a rise in long-term interest rates, as shown in the lowest panel of figure 9.

The simulation results presented in this subsection show that, generally, we cannot exclude either possibility, a not very credible commitment or a fully effective commitment only by observing the response of interest rates to the FOMC's statement, since it is possible that both of the two hypotheses are consistent with the actual observation.

Here is one caveat to our simulation. The simulation result presented above is vulnerable to parameter changes especially for policy reaction function. Figure 10 shows another simulation result with an alternative set of policy reaction parameters. In this simulation, we replaced the standard Taylor rule type coefficients, 0.5 on output gap and 1.5 on inflation by smaller values, 0.2 and 1.1, respectively. This parameter setting implies that the central bank in this economy adopts a weaker feedback rule than the standard Taylor rule. The lower panel reveals that long-term interest rate would fall in contrast to the case in figure 9, which implies that the downward pressure to the long-term interest rates created by the policy actions would dominate the upward pressure stemming from the higher future output gap/inflation.[19] Our guess is that if we are very sure that the Fed adopts such a weak feedback rule as demonstrated in this alternative example, we might be able to say that the observed financial markets' response is more likely to be consistent with a failed commitment, rather than a fully credible commitment.

4 Concluding remarks

The conclusion of the first half of this paper is quite straightforward. Historical federal funds rate levels since 2000 have been close to optimal, given that the optimal inflation forecast targeting framework is sufficiently reliable. Our analysis in the second half leaves a moot argument. Essentially, we do not argue that the commitment/announcement expressed in the FOMC statement of August 12 was perfectly effective or optimal in any sense. Rather, our main finding only shows that observations in the financial markets at that time can be reconciled, at least theoretically, with a simulated market response to an effective monetary policy commitment. The point is that most attempts to derive conclusions regarding the efficacy of monetary policy by examining responses in long-term interest rates are likely to get to a pitfall. Instead, it seems more effective to examine the relationship between the real economic activity, such as household consumption and corporate investment, and monetary policy indicators directly. This would generate more information on the efficacy of policy commitment.

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[19] Actually, the long-term rate started to decline around the first half of September, which might be a consequence of governor Bernanke's September 4 speech referring to the subsequent weakness of the US labor market. If this is the case, to some extent, such decline is naturally attributed to the endogenous Fed's response to weaker economic activity. Some other interpretations are also possible. The most natural view is that just as shown in figure 9-3, long-term rates start to decrease gradually after the peak due to a downturn of the economy as the initial accommodative policy stimulus looses its effect. Another interpretation is, as indicated in the example, the true coefficients in the Fed's policy reaction function eventually proved to be lower than the standard Taylor rule.

A Appendix: Derivation of the optimal policy function

We follow the treatment of the non-negativity constraint as in Watanabe et al. (2001) and apply Kuhn-Tucker conditions in this dynamic optimization problem. Since this problem can be interpreted as a conventional optimal bounded control problem with a linear system, we can set up a Bellman equation with three Lagrange multipliers as follows,

$$\begin{aligned} V\left(y_t,\pi_t\right) &= \min_{i_t}[\frac{1}{2}\left\{y_t^2+\lambda\left(\pi_t-\pi^*\right)^2\right\}-E_t\chi_{t+1}\{(\rho+\alpha\delta)y_t-\delta i_t+\delta\pi_t-y_{t+1}\} \\ &\qquad -E_t\mu_{t+1}(\pi_t+\alpha y_t-\pi_{t+1}) \\ &\qquad -\psi_t i_t \\ &\qquad +\beta E_t V\left(y_{t+1},\pi_{t+1}\right)]. \end{aligned} \tag{14}$$

Careful attention must be paid in writing the signs of Lagrange multipliers χ_t, ψ_t and μ_t. As this is a minimization problem with non-negativity constraint, each sign in front of multiplier must be set so that the multiplier has positive value when the constraint is binding. The first order conditions (FOC) of this problem are

$$E_t\chi_{t+1}\delta-\psi_t = 0 \tag{15}$$

$$E_t\left[\beta y_{t+2}-\alpha\beta\lambda(\pi_{t+2}-\pi^*)-y_{t+1}\right] = E_t\left[\rho\beta\chi_{t+3}-(1+\rho+\alpha\delta)\chi_{t+2}+\frac{1}{\beta}\chi_{t+1}\right] \tag{16}$$

Our interest lies in the explicit form of the optimal reaction function when the constraint is not binding in the current period. Eliminating $E_t y_{t+1}$ and $E_t y_{t+2}$ from FOC by substituting the AS equation yields a second order difference equation of $E_t\pi_{t+1}$ as follows.

$$\beta E_t(\pi_{t+3}-\pi^*)-\left(1+\beta+\alpha^2\beta\lambda\right)E_t(\pi_{t+2}-\pi^*)+E_t(\pi_{t+1}-\pi^*)=\alpha E_t\Psi_t, \tag{17}$$

where $\Psi_t \equiv \left(\rho\beta\psi_{t+2}-(1+\rho+\alpha\delta)\psi_{t+1}+\beta^{-1}\psi_t\right)\delta^{-1}$, and right hand side of eqn(15) rewritten by $E_t\psi_{t+i}$ instead of $E_t\chi_{t+1+i}$. Let $\theta_{1,2}$ be the roots of the characteristic equation, $z^2-\left(1+\beta+\alpha^2\beta\lambda\right)z+\beta=0$. As $\beta>0$ and $\lambda>0$, one root is in the unit circle and the other out of it so that $\theta_1>1$, $\theta_2<1$. Then we can derive the unique solution to this difference equation,

$$\theta_1 E_t(\pi_{t+2}-\pi^*)=E_t(\pi_{t+1}-\pi^*)-\alpha\sum_{i=0}^{\infty}\theta_2^i E_t\Psi_{t+i}. \tag{18}$$

Combining this with eqn(3) yields,

$$E_t y_{t+1}=\left(\frac{1-\theta_1}{\alpha\theta_1}\right)\left(E_t\pi_{t+1}-\pi^*\right)-\frac{1}{\theta_1}\sum_{i=0}^{\infty}\theta_2^i E_t\Psi_{t+i} \tag{19}$$

Note that this equation tells the optimal relationship of $E_t y_{t+1}$ and $E_t \pi_{t+1}$ with the Lagrange multipliers of the non-negativity constraint. Combining equation (19) with the IS equation will leave the optimal reaction function denoted by equation (5).

B Appendix: Collocation method

In this appendix, we explain the numerical algorithm in approximating the value function and optimal policy reaction function in the presence of the non-negativity constraint on nominal interest rates. Specifically, we employ the numerical method known as the collocation method[20] in solving the functional fixed-point problem posed by the Bellman equation.

For convenience, let us restate the Bellman equation (eqn (14)) suppressing the time subscripts as follows,

$$V(\pi, y) = \min_{x \geq 0} \{ f(\pi, y) + \beta E V(g(\pi, y, x, \upsilon, \varepsilon)\}, \tag{20}$$

where $f(\pi, y)$ stands for the period-by-period loss function and $g(\pi, y, x, \upsilon, \varepsilon)$ stands for the state transition function. Note that the nominal interest rate, denoted by x in this appendix, is constrained by the zero lower bound. The state transition function is linear in the state variables and the coefficient matrix is time-invariant, i.e.,

$$g(\pi, y, x, \upsilon, \varepsilon) = \begin{bmatrix} \rho + \alpha\delta & \delta \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} y \\ \pi \end{bmatrix} - \begin{bmatrix} \delta \\ 0 \end{bmatrix} x + \begin{bmatrix} \upsilon \\ \varepsilon \end{bmatrix}.$$

Given the above specification of the Bellman equation and the state transition function, our goal is to interpolate the value function $V(\pi, y)$ in the interval of $-10 \leq \pi \leq 10$ and $-10 \leq y \leq 10$.

The collocation method proceeds in the following steps. First, we discretize the state space by the set of interpolation nodes such that $Node = \{(\pi_{n_\pi}, y_{n_y}) |\ n_\pi = 1, 2, \cdots, N_\pi$ and $n_y = 1, 2, \cdots, N_y\}$.[21] Thus, we discretize the state space into the total of $N_\pi \times N_y$ interpolation nodes. Then we interpolate the value function $V(\cdot)$ using a cubic spline

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[20] For complete elucidation regarding the collocation method, see Judd (1998, Ch.11 and 12) and Miranda and Fackler (2002, Ch.8 and 9).

[21] There are several ways to discretize the state space. One example is Chebychev nodes. However, in order to preserve the exact solution of the value function and optimal policy reaction function at the equally distributed states, equally distributed interpolation nodes have been chosen in this paper.

function[22] over these interpolation nodes as follows.

$$V(\pi_{n_\pi}, y_{n_y}) = \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} c_{ij} \gamma_i^\pi(\pi_{n_\pi}) \gamma_j^y(y_{n_y}) \text{ for each } (\pi_{n_\pi}, y_{n_y}) \in Node. \tag{21}$$

The basis functions $\gamma_i^\pi(\pi_{n_\pi})$ and $\gamma_j^y(y_{n_y})$ take the form of cubic spline functions and are defined as

$$\begin{aligned}
\gamma_i^\pi(\pi_{n_\pi}) &= \left\{ \begin{array}{cc} \frac{2}{3}(1 - 6q_\pi^2(1-q_\pi)) & \text{if } q_\pi = \frac{|\pi_{n_\pi} - \pi_i|}{w} \leq 1 \\ \frac{4}{3}(1-q_\pi)^3 & \text{if } 1 \leq q_\pi \leq 2 \\ 0 & \text{otherwise} \end{array} \right\} \\
\gamma_j^y(y_{n_y}) &= \left\{ \begin{array}{cc} \frac{2}{3}(1 - 6q_y^2(1-q_y)) & \text{if } q_y = \frac{|y_{n_y} - y_j|}{w} \leq 1 \\ \frac{4}{3}(1-q_y)^3 & \text{if } 1 \leq q_y \leq 2 \\ 0 & \text{otherwise} \end{array} \right\},
\end{aligned}$$

where $\pi_i = \underline{\pi} + wi$, where w is an equal step from the lower bound of state π (which is -10 in this paper) to the upper bound (which is 10 in this paper). The definition of y_j is similar. Interpolation equations (21) could be expressed compactly using the tensor product notation as follows,

$$\mathbf{v} = [\mathbf{\Gamma}_\pi \otimes \mathbf{\Gamma}_y] \cdot \mathbf{c}, \tag{22}$$

where $\mathbf{v}$ stands for $N_\pi N_y \times 1$ vector of the values of $V(\pi_{n_\pi}, y_{n_y})$ for each interpolation node, $\mathbf{\Gamma}_\pi$ stands for $N_\pi \times N_\pi$ matrix of the basis functions $\gamma_i^\pi(\pi_{n_\pi})$ (i.e., each matrix element is defined as $\mathbf{\Gamma}_\pi[i, n_\pi] = \gamma_i^\pi(\pi_{n_\pi})$), Γ_y stands for $N_y \times N_y$ matrix of the basis functions $\gamma_j^y(y_{n_y})$, and $\mathbf{c}$ stands for $N_\pi N_y \times 1$ vector of the basis coefficients c_{ij}.

Next, we turn to the right-hand side of the Bellman equation (20). In approximating the expected value function, i.e., $E[V(g(\pi, y, x, \upsilon, \varepsilon)]$, we assume the distribution of the error terms (υ, ε) to be *i.i.d.* multivariate normal. Under the assumption of normal distribution, the expected value function can be approximated by the Gaussian-Hermite quadrature method[23] – a member of the Gaussian quadrature methods which is

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[22] There are several other options for the basis function. One of the most frequently used basis functions is the Chebychev polynomial, which is known to possess superior properties when the curvature of the function to be interpolated is "nice and smooth." In contrast, the cubic spline function is known to possess superior properties when the function contains some "kinks." Since the value function and the optimal policy reaction function are kinked due to the presence of the zero lower bound in this paper, the cubic spline function will be our choice as a basis function. For more details regarding the cubic spline interpolation, see Judd (1998, Ch.6), Cheney and Kincaid (1999), and Miranda and Fackler (2002, Ch.6).

[23] For more details regarding the Gaussian Quadrature method, see Judd (1998, Ch.7) and Miranda and Fackler (2002, Ch. 5).

specifically used when the error terms are normally distributed. The Gaussian-Hermite quadrature method discretizes the random space with the set of quadrature nodes such that $QNode = \{(\upsilon_{h_\upsilon}, \varepsilon_{h_\varepsilon}) | h_\upsilon = 1, 2, \cdots, M_\upsilon \text{ and } h_\varepsilon = 1, 2, \cdots, M_\varepsilon\}$ with corresponding quadrature weights $\omega_{h_\upsilon h_\varepsilon}$. Thus, we discretize the random space into a total of $M_\upsilon \times M_\varepsilon$ quadrature nodes. Then by substituting the interpolation equation (21) for the value function $V(g(\pi, y, x, \upsilon, \varepsilon)$, the right-hand side of the Bellman equation can be approximated as

$$RHS_{n_\pi n_y}(\mathbf{c}) = \min_{x \geq 0} \left\{ f(\pi_{n_\pi}, y_{n_y}) + \beta \sum_{h_\upsilon=1}^{M_\upsilon} \sum_{h_\varepsilon=1}^{M_\varepsilon} \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} \omega_{h_\upsilon h_\varepsilon} c_{ij} \gamma_{ij}(g(\pi_{n_\pi}, y_{n_y}, x, \upsilon_{h_\upsilon}, \varepsilon_{h_\varepsilon})) \right\} \tag{23}$$

for each $(\pi_{n_\pi}, y_{n_y}) \in Node$, where γ_{ij} stands for the cross products of the basis function. The minimization of the above problem with respect to x can be attained using a standard Quasi-Newton optimization method. It should be noted that when implementing this minimization problem, one should pay attention to the corner solution of the minimization problem due to the zero lower bound constraint on the control variable x.

Finally, by equating equation (21) and equation (23) for each interpolation node, we obtain the following approximation of the Bellman equation (20);

$$\sum_{i=1}^{N_\pi} \sum_{j=1}^{N_y} c_{ij} \gamma_i^\pi(\pi_{n_\pi}) \gamma_j^y(y_{n_y}) = RHS_{n_\pi n_y}(\mathbf{c}) \quad \text{for each } (\pi_{n_\pi}, y_{n_y}) \in Node. \tag{24}$$

Using the tensor product notation, the above equation can be compactly expressed as

$$[\mathbf{\Gamma}_\pi \otimes \mathbf{\Gamma}_y]\mathbf{c} = \mathbf{RHS}(\mathbf{c}), \tag{25}$$

where $\mathbf{RHS}(\mathbf{c})$ stands for $N_\pi N_y \times 1$ vector of the values of $RHS_{n_\pi n_y}(\mathbf{c})$. Now the task is to find the unknown basis coefficient vector $\mathbf{c}$ from the above nonlinear equation system (25). The nonlinear equation system can be solved using an iterative nonlinear root-finding technique such as the Functional Iteration method, Newton's method or a Quasi-Newton method.[24] For computational ease, we adopt the Functional Iteration method as the solution algorithm.

Algorithm 2 *(Functional Iteration method)*
Step 1: Choose the degree of approximation N_π, N_y, M_υ, and M_ε. Then set the

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[24] For more details regarding the nonlinear root-finding technique, see Judd (1998, Ch.5) and Miranda and Fackler (2002, Ch.3).

appropriate interpolation nodes and quadrature nodes for the state space and random space, respectively. Guess the initial basis coefficients vector $\mathbf{c}_0$.
Step 2: Update the basis coefficient vector by the following functional iteration;

$$\mathbf{c}_{k+1} \leftarrow [\mathbf{\Gamma}_{\pi}^{-1} \otimes \mathbf{\Gamma}_{y}^{-1}] \cdot \mathbf{RHS}(\mathbf{c}_k).$$

Step 3: Check for convergence. If $|c_{ij,k+1} - c_{ij,k}| < \tau$ *for any* i *and* j, *where* τ *is a convergence tolerance parameter, then stop. Otherwise, repeat step 2.*

Once convergence has been reached, the interpolation of the value function $V(\pi, y)$ is now attained. Of course, as a by-product of interpolating the value function, the approximation of the optimal policy function $x^*(\pi, y)$ will also be attained at the same time. It should be noted that one can attain the desired level of approximation by controlling the degree of interpolation nodes, quadrature nodes and convergence tolerance parameter τ with a trade-off of convergence speed.[25]

C Appendix: GMM estimation results

This appendix presents the details of estimation of the parameter values used in the simulation analysis.

Here are two models estimated using the actual US quarterly data for recent two decades. Note that in addition to theoretical illustrations presented in the main text, these estimated models include some exogenous terms, such as real exchange rates denoted as $\sum \omega_i z_{t-i}$ and real federal government expenditure κg_t to control various disturbance to the real economy.

- **Model 1: Backward-looking economy**

$$\begin{aligned} y_t &= \rho y_{t-1} - \delta\left(i_{t-1} - E_{t-1}\pi_t\right) + \kappa g_t + \sum_{i=1}^{3} \omega_i z_{t-i} + \upsilon_t \\ \pi_t &= \gamma \pi_{t-1} + \alpha y_{t-1} + \varepsilon_t. \end{aligned}$$

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[25] In our paper, we have set the parameter values as follows; $N_\pi = 20$, $N_y = 20$, $M_\upsilon = 3$, $M_\varepsilon = 3$ and $\tau = 10^{-8}$. With these parameter values, the maximum absolute approximation error of the value function was smaller than 10^{-3}. Using the Pentium III computing environment, convergence was attained within 10 minutes in most cases.

- **Model 2: Hybrid forward-looking economy**

$$\begin{aligned} y_t &= \rho y_{t-1} + (1-\rho)E_t y_{t+1} - \delta(i_t - E_t\pi_{t+1}) + \kappa g_t + \sum_{i=1}^{3}\omega_i z_{t-i} + \upsilon_t \\ \pi_t &= \gamma\pi_{t-1} + (1-\gamma)\,E_t\pi_{t+1} + \alpha y_t + \varepsilon_t. \end{aligned}$$

Data source is as follows. Output gap is calculated using Hodrick-Prescott filter with the smoothing parameter 4000. Real effective exchange rate is released by Board of Governors and inflation is core CPI index y-o-y rates. All those parameter values are estimated simultaneously using multiple GMM. Estimation results are presented in the following table.

Table 2: Multiple GMM estimation

	model 1	model 2
ρ	0.750	0.420
	(0.18)	(0.12)
δ	0.448	0.136
	(0.41)	(0.21)
γ	0.912	0.688
	(0.03)	(0.20)
α	0.104	0.143
	(0.03)	(0.09)
κ	0.728	0.283
	(0.45)	(0.19)
ω_1	0.579	0.227
	(0.76)	(0.27)
ω_2	-0.048	-0.100
	(0.83)	(0.19)
ω_3	-0.593	-0.163
	(0.50)	(0.23)
nma	2	4

Note1: sample period 1980Q2-2001Q4.

Note 2: Instrumental variables are energy price index, the Federal defense expense, output gap(-2) the federal funds rate(-2) and real effective exchange rate

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Figure 1

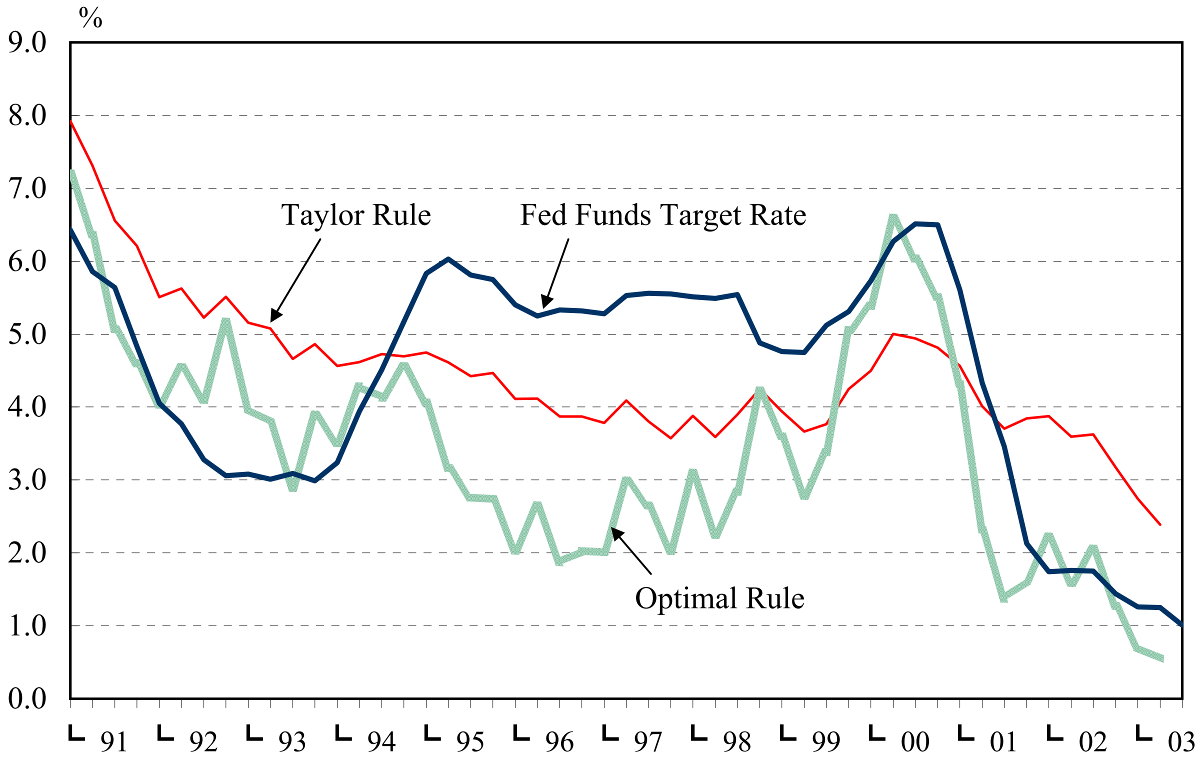
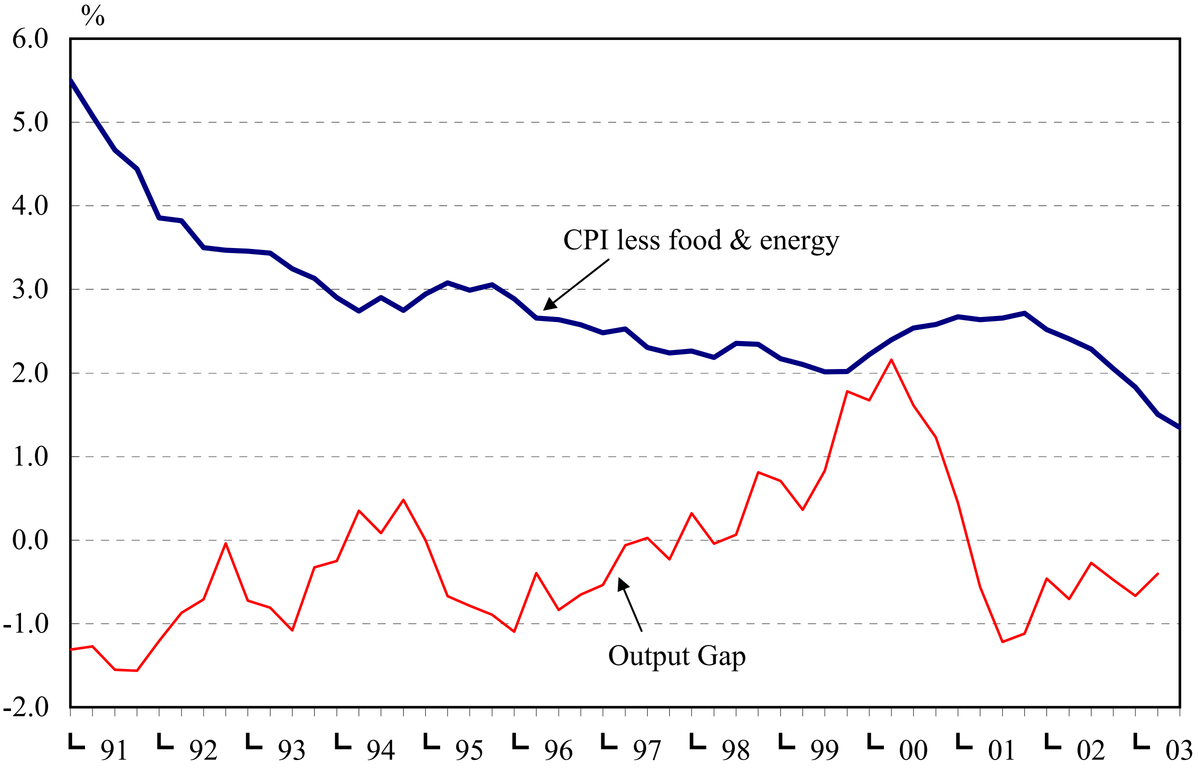


Figure 2



Responses to a discretionary interest rate cut: A benchmark case

Figure 3-1 : Output gap

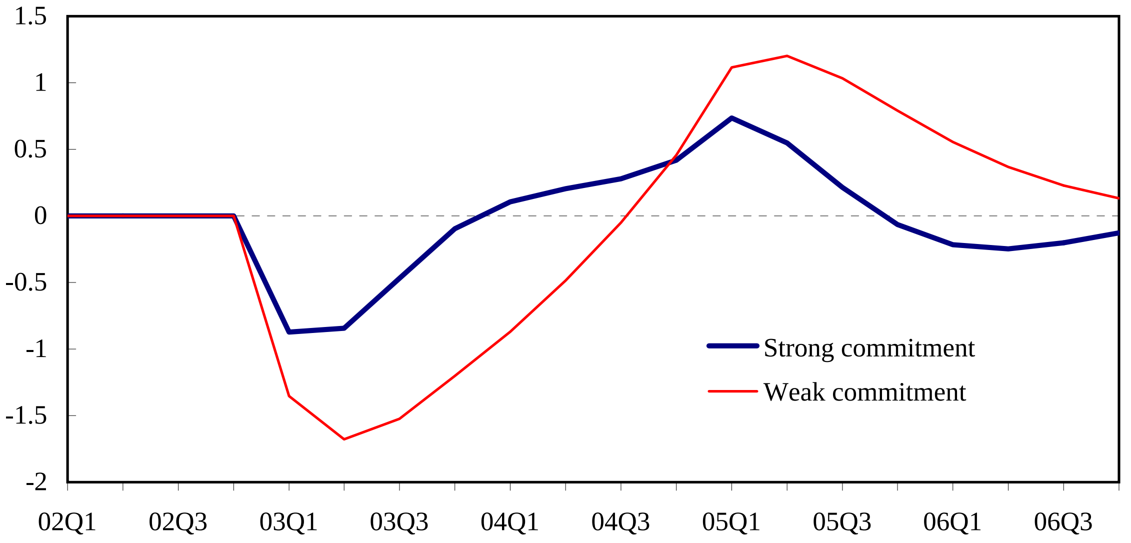


Figure 3-2 : Federal funds rate

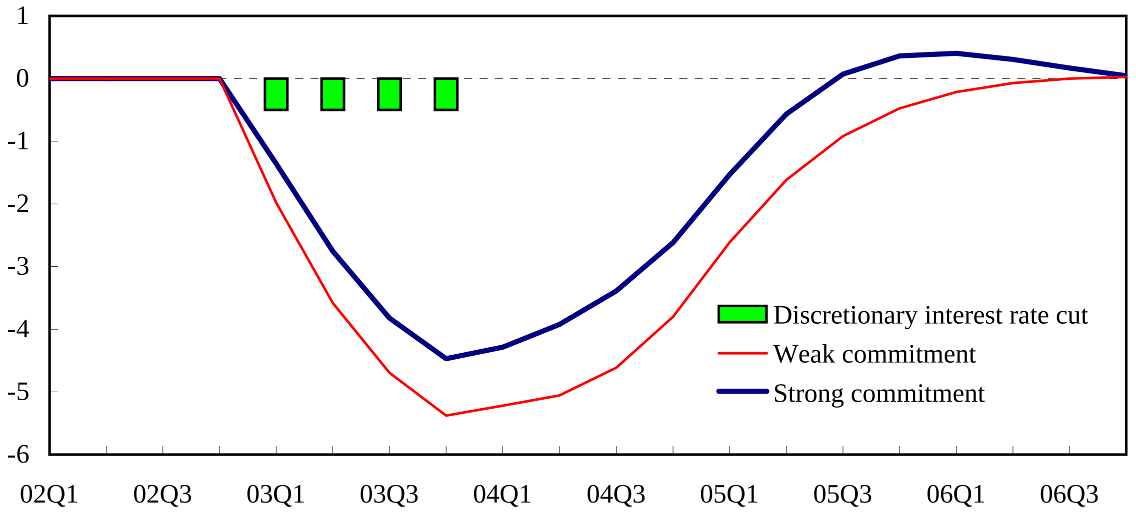
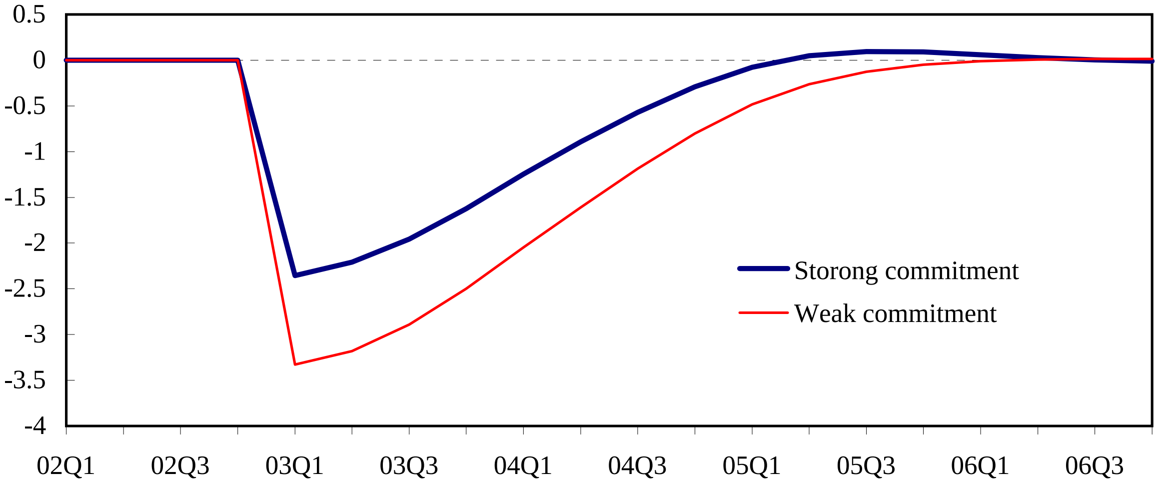


Figure 3-3 : Long-term rate



The FOMC mentioned an "unwelcome fall in inflation" (May 6)

Figure 4-1

Changes in Implied Forward Rates (1-year)

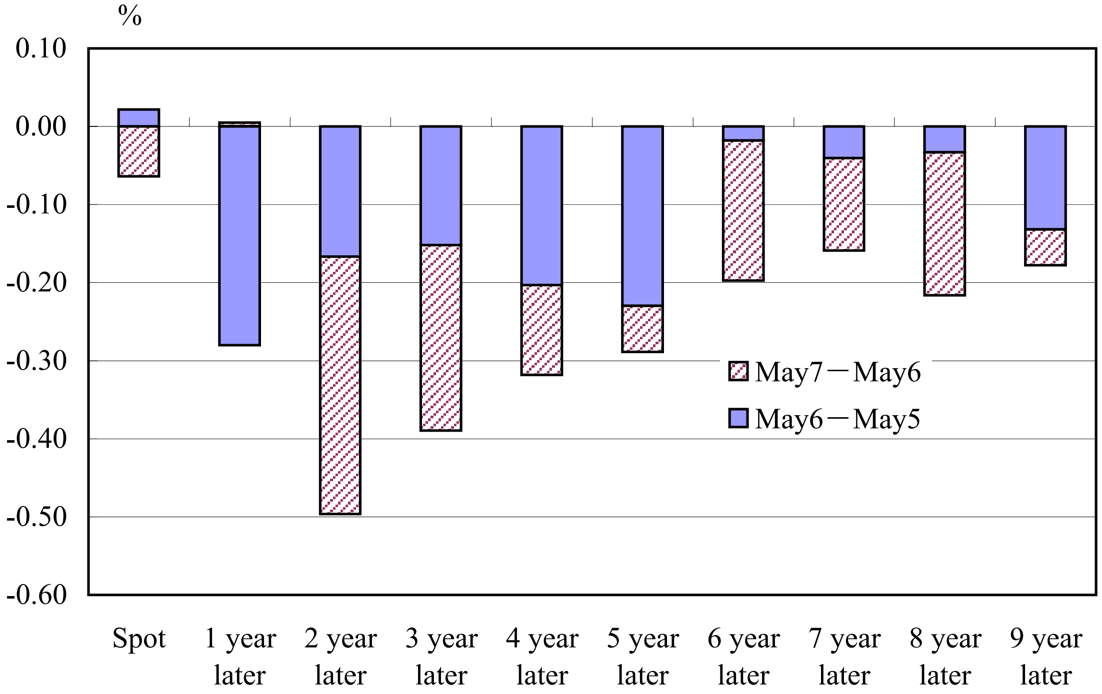
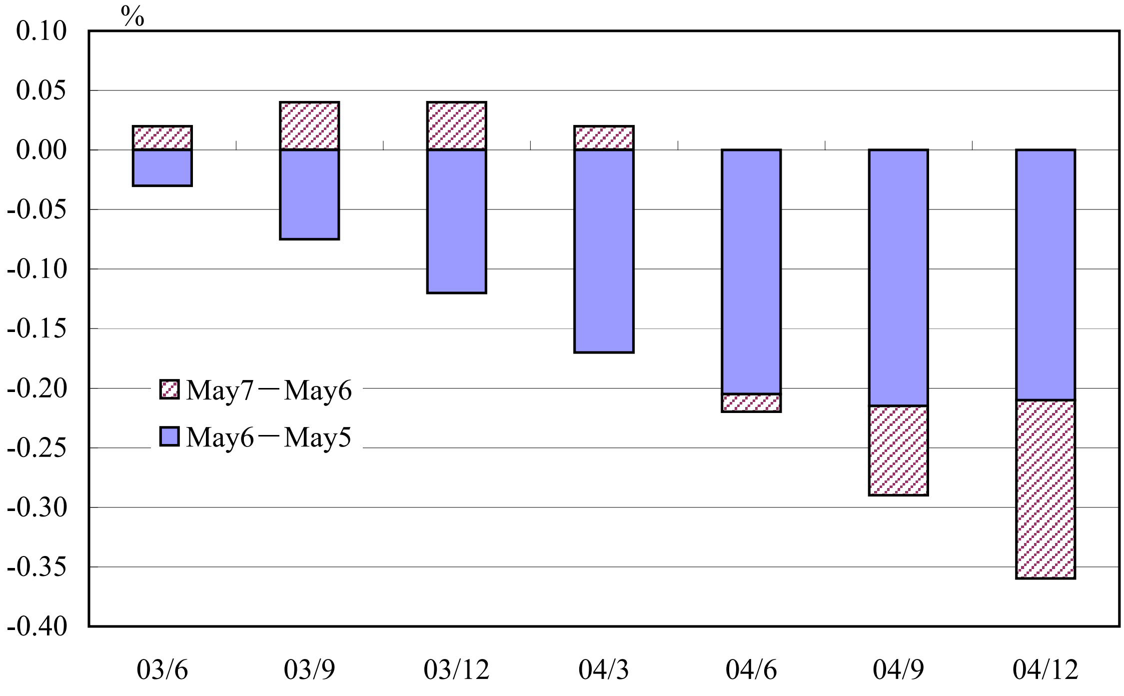


Figure 4-2

Changes in Eurodollar Interest Rates Futures (3-Month)



The FOMC decided to lower the target rate by 25 basis points (June 25)

Figure 5-1

Changes in Implied Forward Rates (1-year)

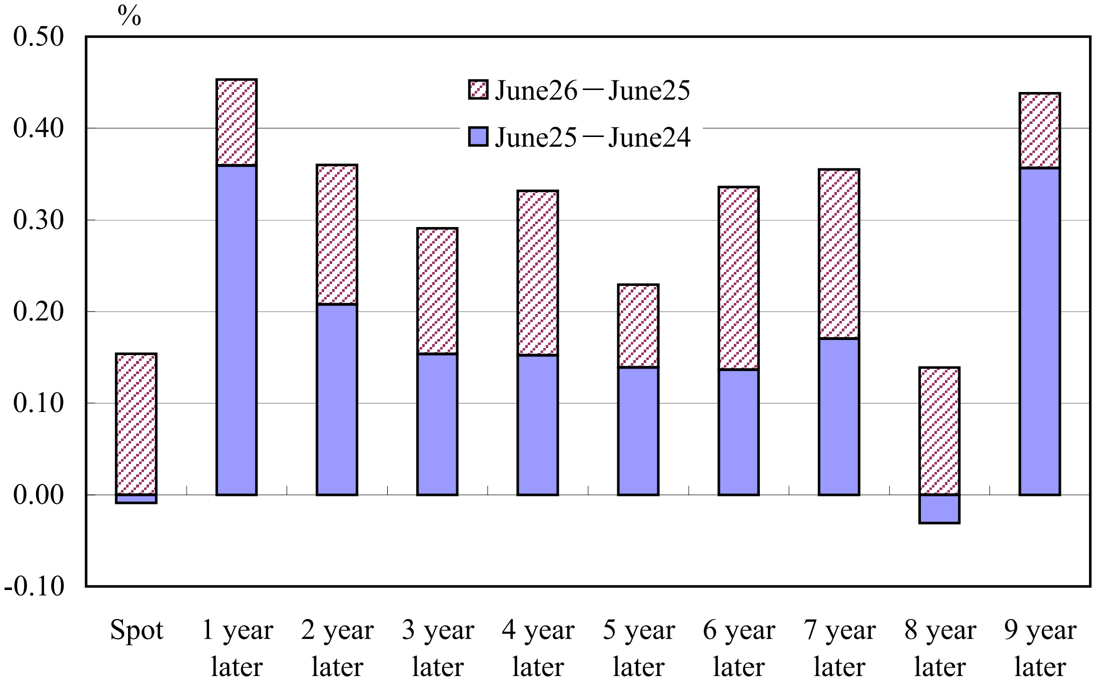
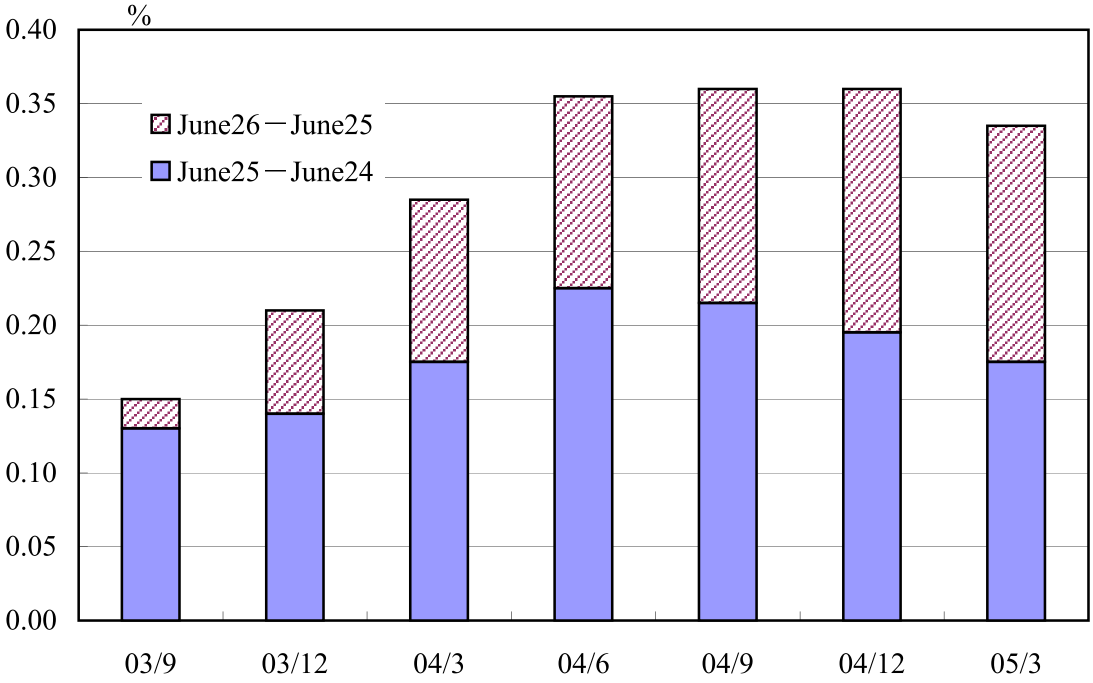


Figure 5-2

Changes in Eurodollar Interest Rates Futures (3-Month)



The FOMC mentioned the maintainance of policy accommodation for a considerable period (August 12)

Figure 6-1

Changes in Implied Forward Rates (1-year)

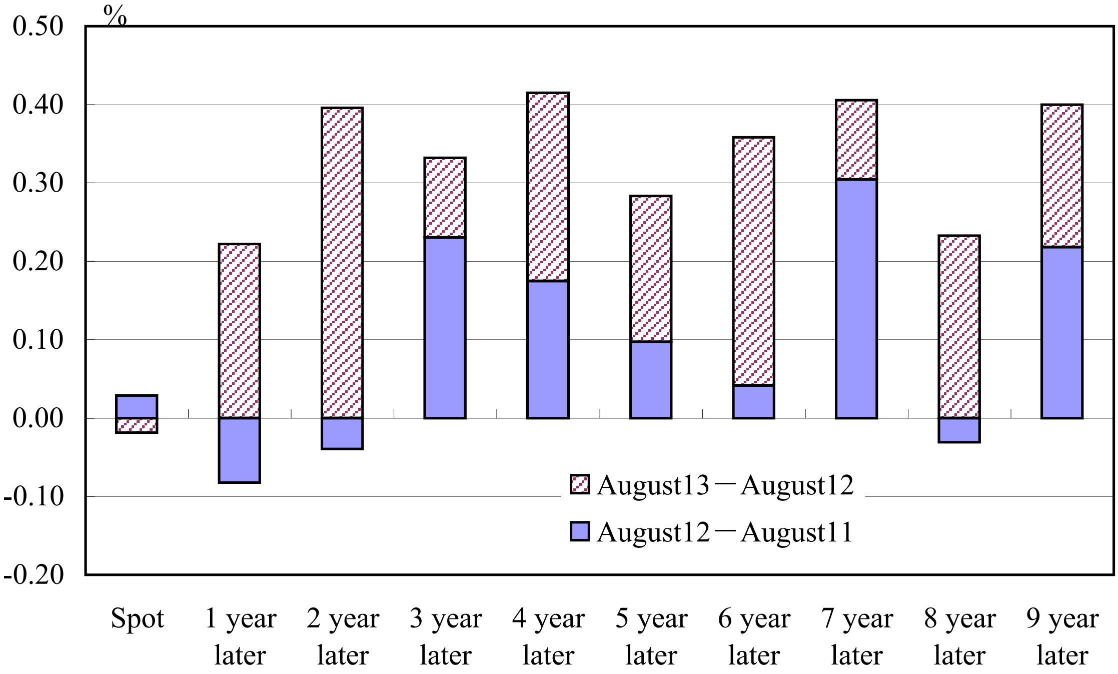
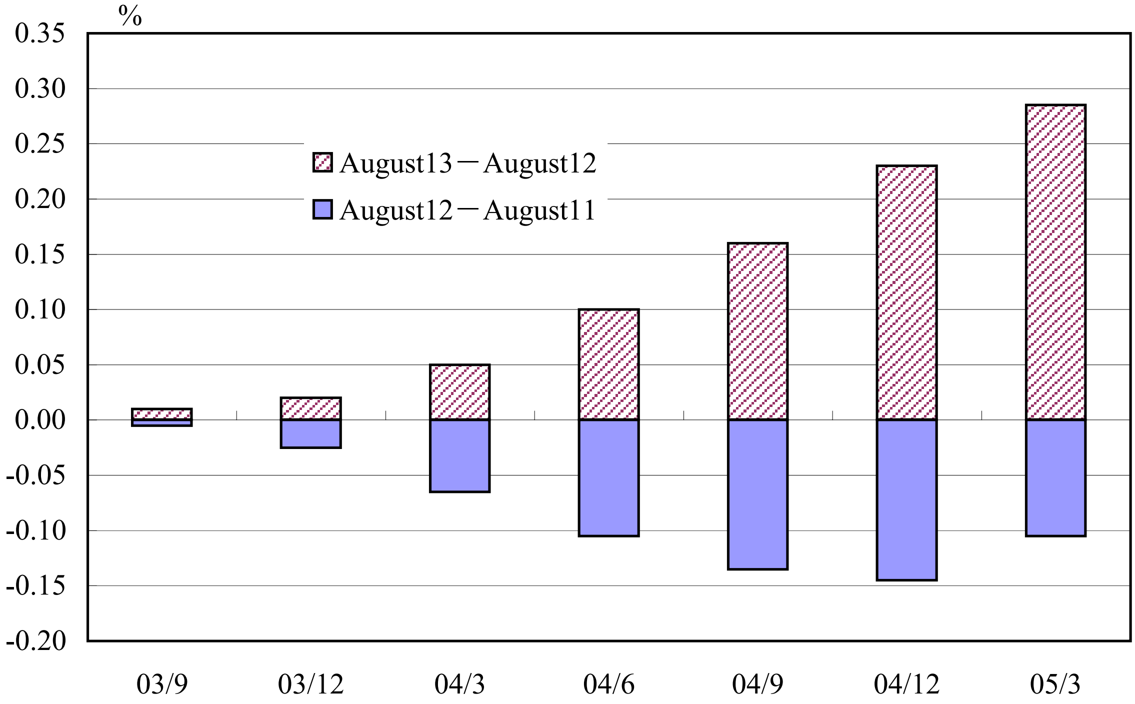


Figure 6-2

Changes in Eurodollar Interest Rates Futures (3-Month)



Simulated responses to an announcement of "unwecome fall in inflation''

Figure 7-1 : Output gap

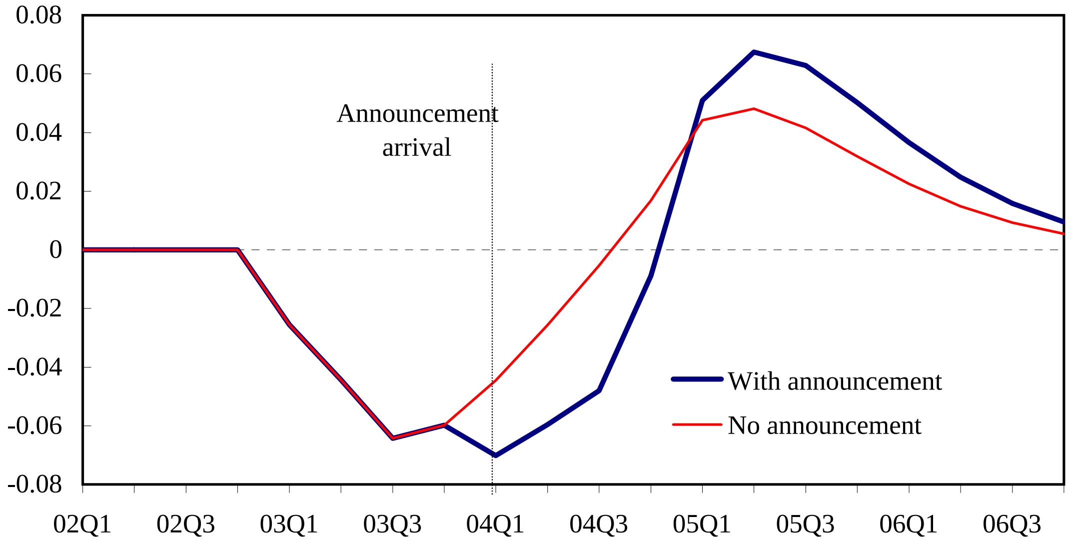


Figure 7-2 : Inflation

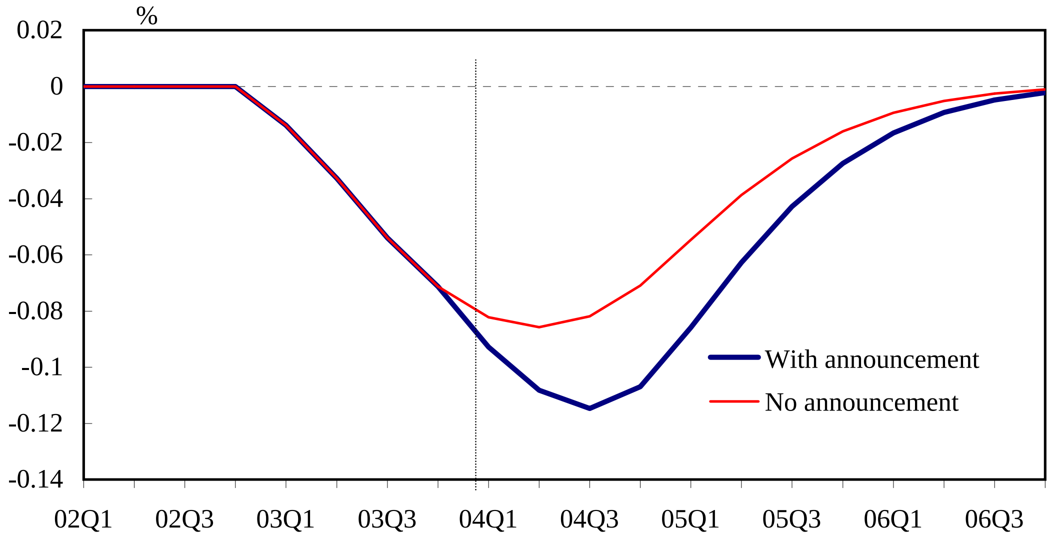
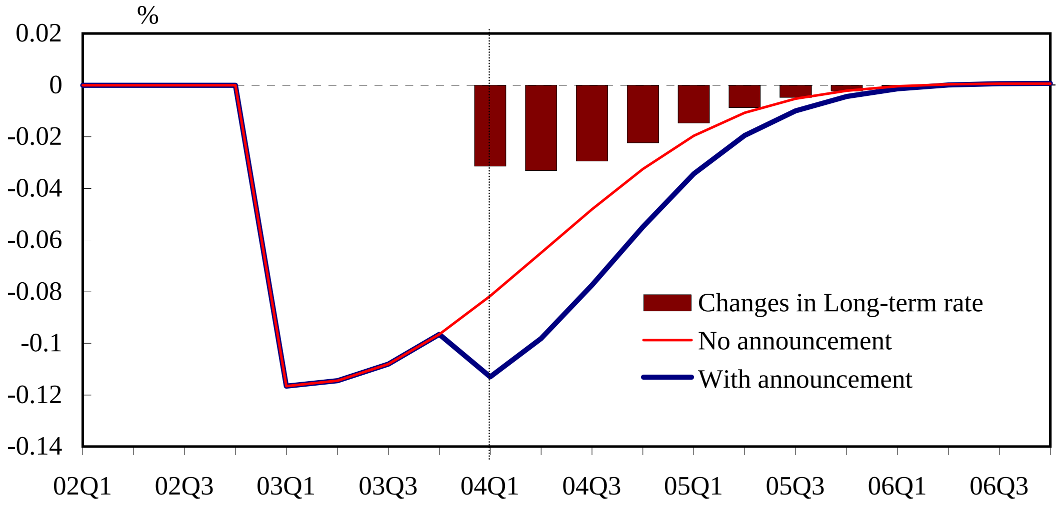


Figure 7-3 : Long-term rate



Simulated responses to an unanticipated rise in federal funds rates

Figure 8-1 : Output gap

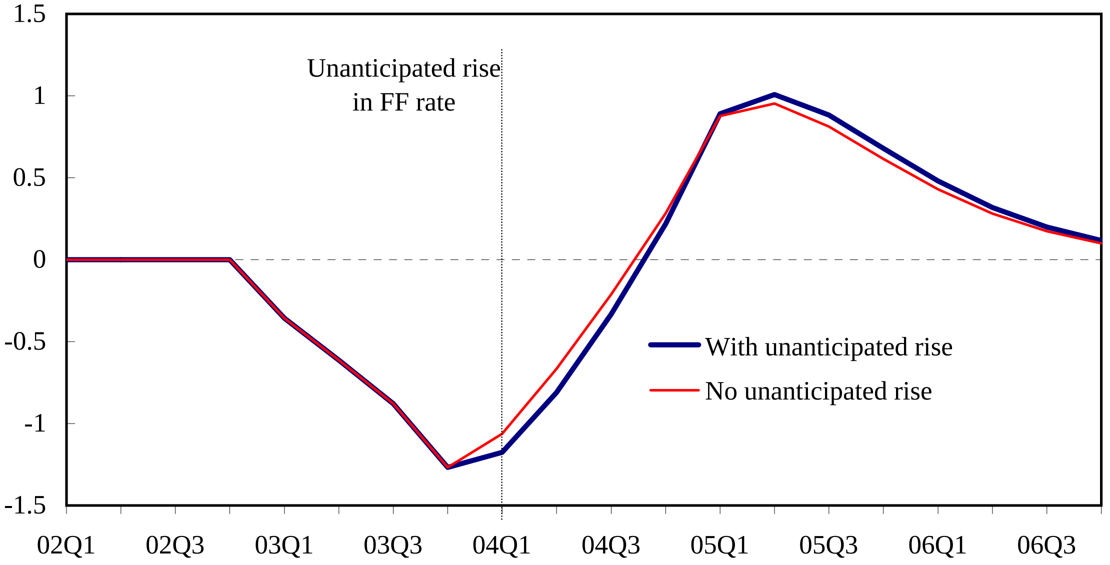


Figure 8-2 : Federal funds rate

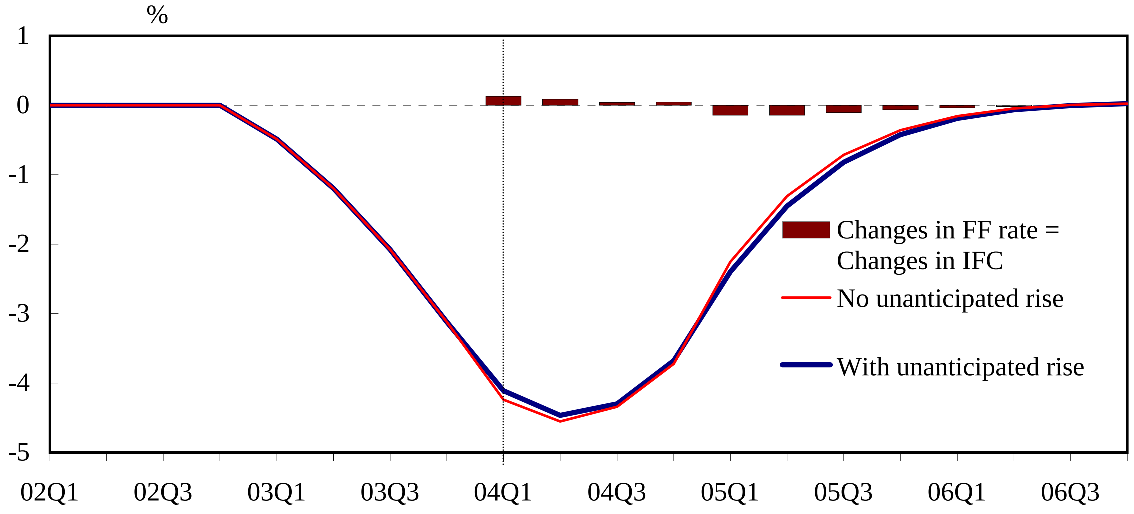
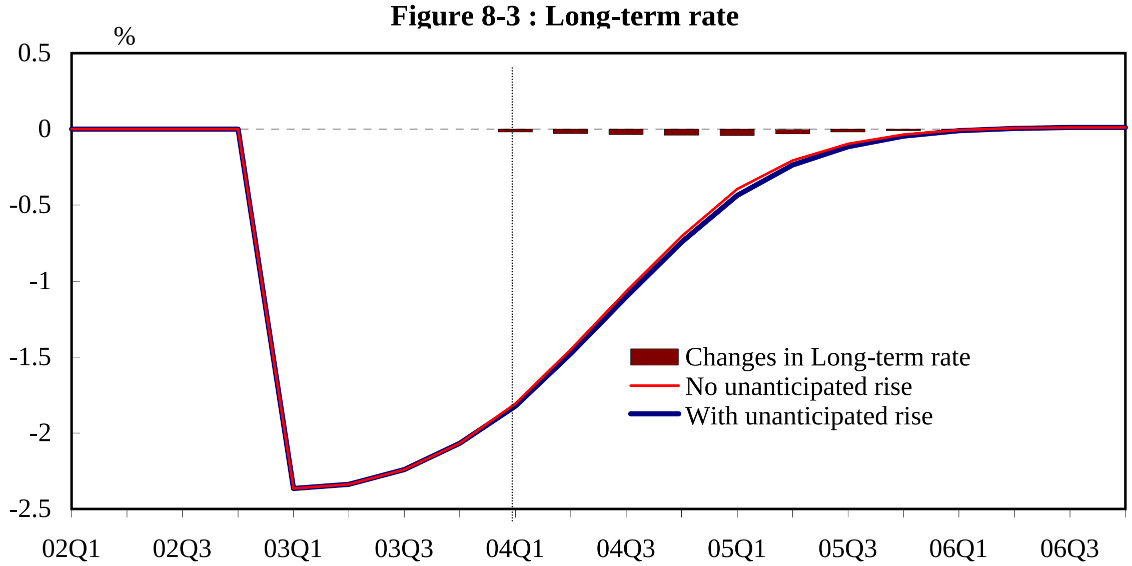


Figure 8-3 : Long-term rate



Simulated responses to a change in the dgree of commitment (1)

Figure 9-1 : Output gap

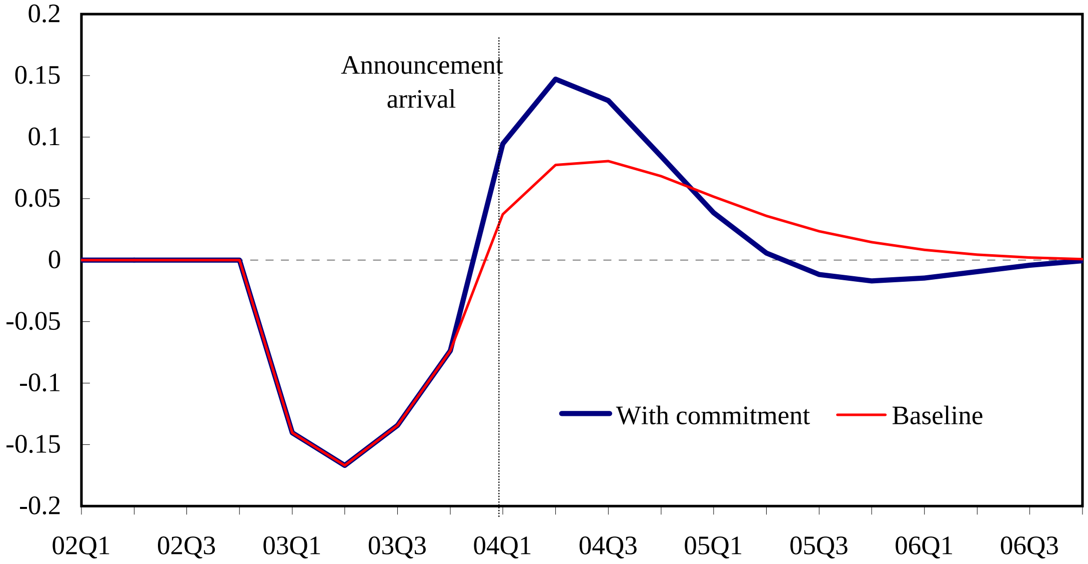


Figure 9-2 : Federal funds rate

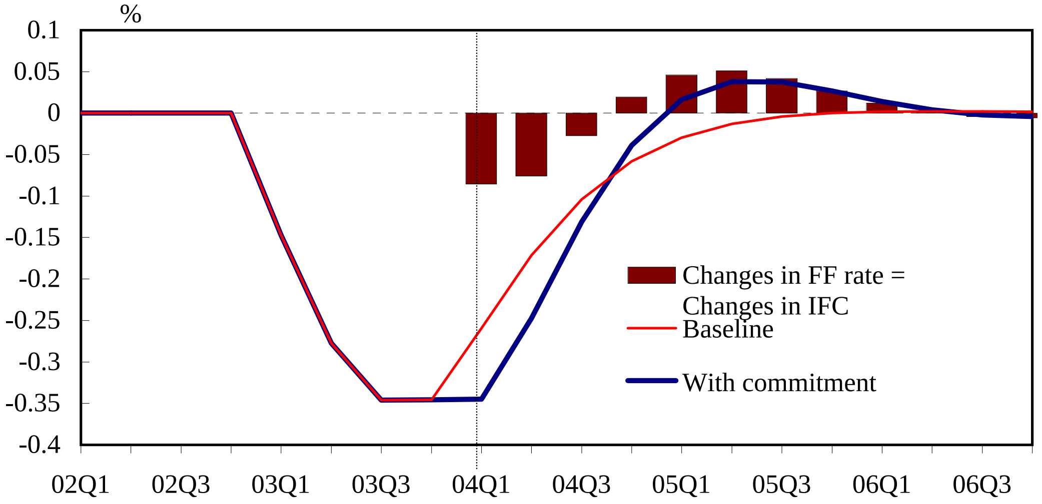
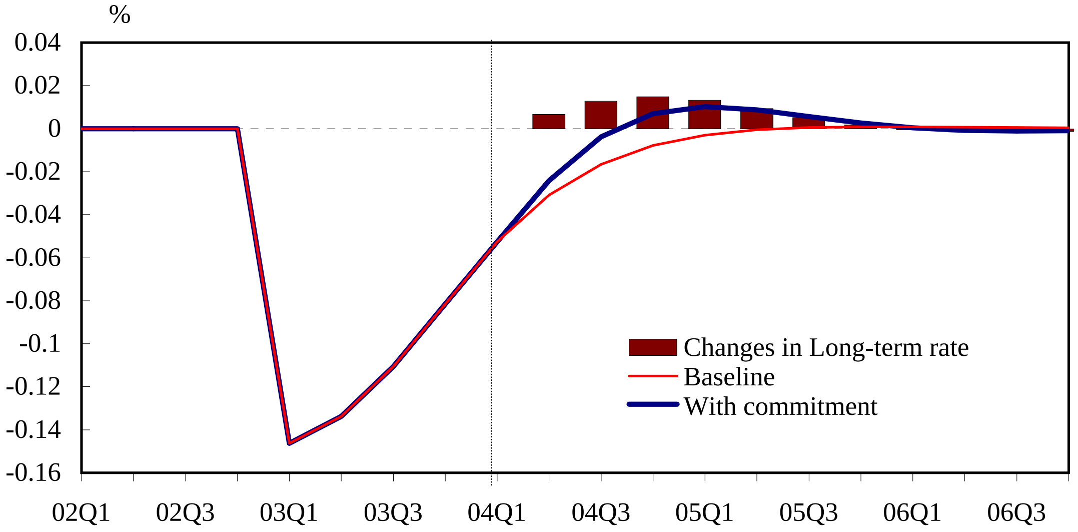


Figure 9-3 : Long-term rate



Simulated responses to a change in the dgree of commitment (2)

Figure 10-1 : Output gap

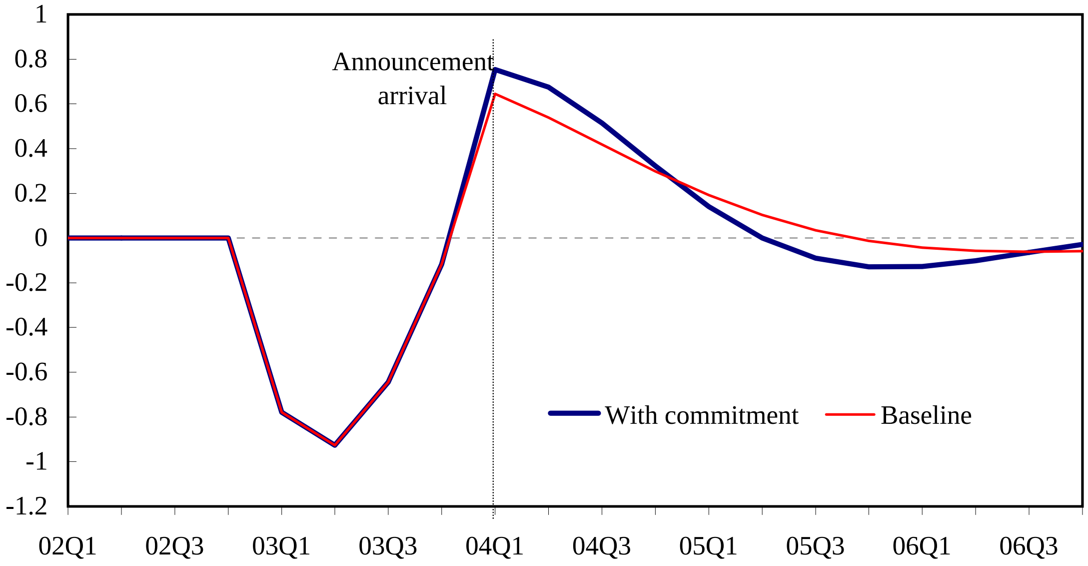


Figure 10-2 : Federal funds rate

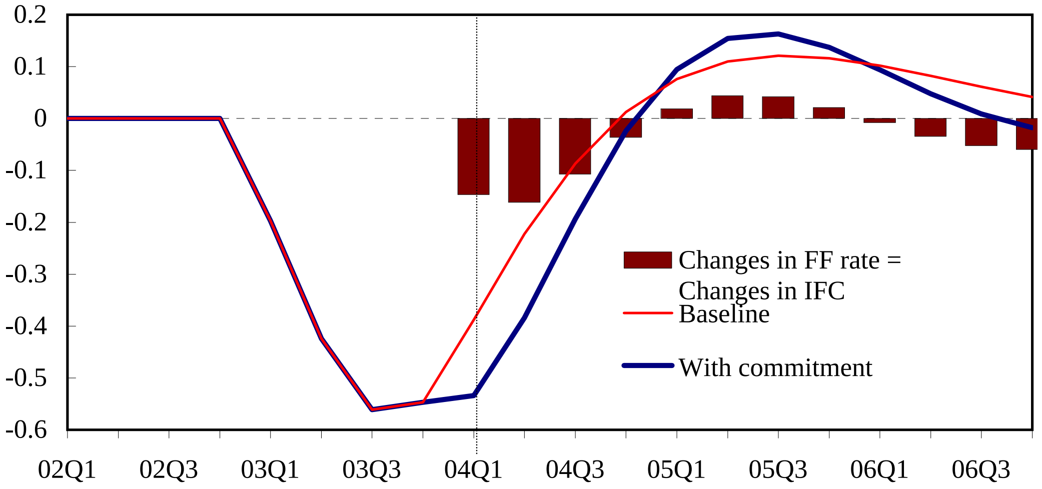


Figure 10-3 : Long-term rate

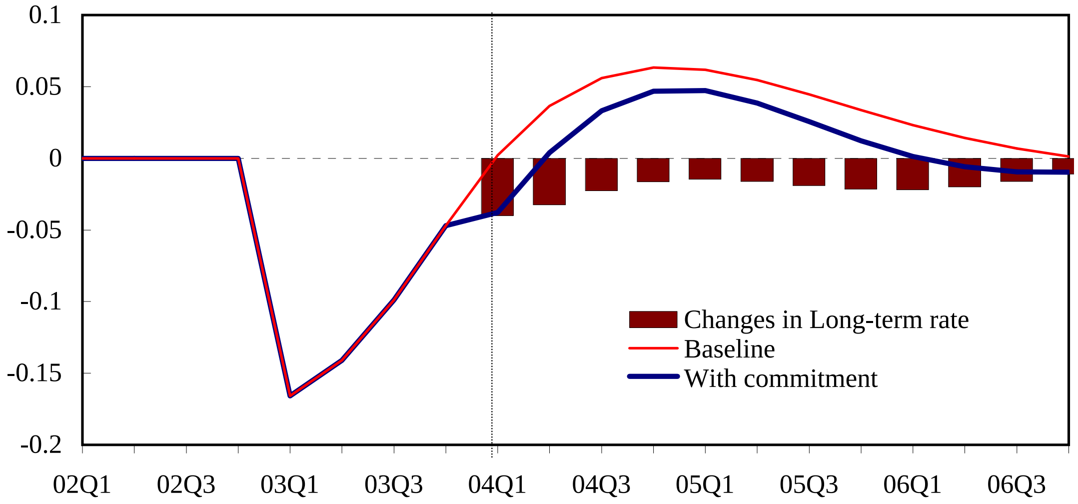


Figure 11

Treasury Bond Yeild