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Investment with Uncertainty: Detection of Decomposed Uncertainty Factors Affecting Investment

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Abstract

We empirically analyze the effects of uncertainty on investment, by types of uncertainty and by properties of stochastic processes; that is, permanent and transitory components separated from economic variables. According to our results, based on Japanese economic data, we find significant negative effects of uncertainty. As for types of uncertainty, uncertainty in the transitory component of the exchange rate has significant negative effects throughout the estimation period, and the uncertainty in the permanent component of the exchange rate had significant negative effects until the 1980s. In the late 1990s, the uncertainty in the permanent component of total debts of failed firms indicates significant negative effects.

Keywords: investment nonlinearity, Tobin's q, uncertainty, Markov switching, MCMC

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1 Introduction

The Japanese economy has experienced several long-lasting recessions since the 1990s, under a continuously easy monetary policy. Although several factors, such as zero bound on the nominal interest rate and the delay

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in carrying out necessary structural changes,why economic recovery has been elusive. The investment slump incurred by future uncertainty may also explain the long-lasting recession. When there is uncertainty, even if circumstances may suggest investers with expectation of adequate returns, firms have incentives to postpone their investment plans. [1]

The model presented by Dixit and Pindyck (1994) has three important assumptions: the future profitability is unknown, this uncertainty is partially resolved by additional information, and the timing of investment is flexible. Based on these assumptions, Dixit and Pindyck (1994) argued that future uncertainty can negatively affect investment incentives. Meanwhile, Hartman (1972) and Abel (1983) also suggest that increased uncertainty could raise investment based on the assumption of its marginal profitability of capital stock. After all, the relationship between investment and uncertainty depends on actual parameters with the economy. [2]

In this paper, to analyze some accounting for the long-lasting recessions of the Japanese economy, we are interested in the negative effects of uncertainty on investment by using Japanese economic data, altough we analize both cases of over-investment and under-investment. Our research has the following three characteristics: (i) Searching various exogenous fundamental uncertainty that affects investment, such as exchange rates (not the integrated uncertainty including endogenous efforts such as profits and stock prices). [3];

(ii) Extracting permanent and transitory components from variables and using them to analyze their effects on investment, since it is theoretically suggested that the effects of these different components could have different effects on investment; [4];

(iii) Estimating and testing the negative effects of various possible uncertainty on investment by the Markov switching model and MCMC sampling, since theoretical analysis suggests that the effects of uncertainty could be nonlinear and that not all information is necessarily known.

It is important to figure out which uncertainty appears and when it appears to resolve the negative effects of the uncertainty on investment. As far as we know, however, there is little previous research on this field, and our research should contribute substantially to this field.

Based on the results of our analyses, several investment slumps in the past were significantly affected by

[1] Theoretical analyses of investment under uncertainty have been intensively studied over past decade by the importation of ideas from finance. If investment is irreversible (or if the value to vend is less than that to purchase), there is option value on waiting. This approach is called the real option approach. See Dixit and Pindyck (1994) for details.

[2] See Pindyck and Salimano (1993), Leahy and Whited (1996) for empirical studies of investment under uncertainty. See Ogawa and Suzuki (2000) for empirical studies using Japanese data.

[3] See the Appendix for uncertainties of profits and stock prices are argued.

[4] See Corrado and Holly (2001) about the effects of permanent and transitory components on investment.

uncertainty. We also find specific uncertainty during specific periods. Throughout the estimation period, uncertainty of the transitory component of the exchange rate has played an important role, and uncertainty of the permanent component of the exchange rate has also affected the slump of investment until the 1980s. After the late 1990s, the bankruptcy variable has shown remarkable negative effects on investments.

In Section 2, we analyze the effects of uncertainty on investment and point out some theoretical features of irreversible investment. In Section 3, based on the features analyzed in Section 2, we will verify the effects of uncertainty on investment by components and by time series properties using the Monte Carlo approach to estimate time-varying and nonlinear parametric models. Section 4 concludes the paper.

2 Theoretical Features of Irreversible Investment Model

In this section, we point out important features of the effects of uncertainty on investment based on the framework of Dixit and Pindyck (1994) and Corrado and Holly (2001).

First, we assume the profit function as

$$\Pi(K,\theta) = A\theta K, \tag{1}$$

where Π is profit, K is the stock of capital, A is a parameter that represents technological progress. Labor is left out of this model to simplify the argument. We suppress the time subscript hereafter unless it is necessary, to avoid confusion. The stochastic term θ is an aggregate shock that is assumed to follow a geometric mean-reverting process without drift:

$$d\theta = \mu(\bar{\theta} - \theta)\theta dt + \sigma\theta dz, \tag{2}$$

where $\bar{\theta}$ is the equilibrium level of the fundamentals, z is a Winner process with $E[dz] = 0$, $E[(dz)^2] = dt$, and the expected value of the changing rate of θ ($d\theta/\theta$) is $\mu(\bar{\theta} - \theta)dt$ and its variance is $\sigma^2 dt$. This stochastic process will be a random walk (Winner process) when $\mu = 0$, and the degree of mean reversion increases as μ increases.

The firm undertakes gross investment I and incurs depreciation at a constant rate $\delta \geq 0$. Thus the change in capital stock follows the process

$$dK = (I - \delta K)dt, \tag{3}$$

The adjustment costs of investment are assumed to be a strictly convex and can be written as [5]

$$C(I) = \begin{cases} c_{21}I + (1/2)c_3I^2 & if\ I > 0 \\ 0 & if\ I = 0 \\ c_{22}I + (1/2)c_3I^2 & if\ I < 0 \end{cases},$$

When the coefficient of the first term is $c_{21} > c_{22}$, investment is asymmetric. When $c_{21} > c_{22} = 0$, investment becomes perfectly irreversible. We assume that the representative firm maximizes its expected present value of cash flow under the given discount rate $r > 0$. Then, the value of the firm at time t is

$$V(K,\theta) = \max_{I_t} E_t \int_t^{\infty} [\Pi(K_s,\theta_s) - c(I_s)]e^{-r(s-t)}ds. \tag{4}$$

The present value satisfies the following Bellman equation:

$$rV(K,\theta) = \max_I \left[\Pi(K,\theta) - C(I) + \frac{EdV}{dt}\right]. \tag{5}$$

The equation suggests that the opportunity cost $rV(K,\theta)$ equals the sum of the instantaneous cash flows and the expected capital gain. The expected capital gain is obtained by applying Ito's lemma,

$$\frac{EdV}{dt} = (I - \delta K)V_k + \mu(\bar{\theta} - \theta)\theta V_\theta + \frac{1}{2}\sigma^2\theta^2 V_{\theta\theta}. \tag{6}$$

This equation indicates that the expected capital gain depends on the marginal valuation of a unit of installed capital V_k. Now define $q \equiv V_k$, which is the shadow value of installed capital. By using this shadow value, the Bellman equation can be written as [6]

$$rV = \max_I \left\{AK\theta - C(I) + (I - \delta K)q + \mu(\bar{\theta} - \theta)\theta V_\theta + \frac{1}{2}\sigma^2\theta^2 V_{\theta\theta}\right\}. \tag{7}$$

Then optimal investment solves the term

$$\max\{Iq - C(I)\}. \tag{8}$$

Via the first order conditions, we obtain

$$\begin{aligned} I_1 &= \left(\frac{q - c_{21}}{c_3}\right) \qquad for\ I > 0 \\ I_2 &= \left(\frac{q - c_{22}}{c_3}\right) \qquad for\ I < 0. \end{aligned} \tag{9}$$

[5] To solve the second difference equation easily, we assume constant term = 0 and the coefficients of the second term are equal. This simplification does not affect our main results.

[6] In this paper, revision costs of newly installed capital is fixed as 1. Thus, q becomes famous Tobin's q.

Since the term $Iq - C(I)$ is zero for zero investment, $I_i, (i = 1, 2)$ is only optimal if $I_i q - C(I_i) \geq 0, (i = 1, 2)$. Therefore we define q_u and q_l as the unique roots of $I_1 q - C(I_1) = 0$ and $I_2 q - C(I_2) = 0$, respectively. Then, we obtain the optimal investment schedule

$$I^* = \begin{cases} I_1 & if\ q > q_u \\ 0 & if\ q_l \leq q \leq q_u \\ I_2 & if\ q < q_l \end{cases},$$

This suggests that the investment schedule of the representative firm undergoes apparent regime switching. q_l and q_u represent upper thresholds of inaction range and lower thresholds, respectively. By solving this model for the shadow capital value q, we obtain [7]

$$q_u(\theta) = \frac{1}{r + \delta - \mu\bar{\theta}} A\theta + B\_\theta^{\lambda_-} H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_-; b\right)\ if\ \theta \in [\theta_u, \infty), \tag{10}$$

$$q_l(\theta) = B_+ \theta^{\lambda_+} H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_+; b\right)\ if\ \theta \in [0, \theta_l], \tag{11}$$

where θ_l, θ_u are the lower and the upper thresholds of inaction range, respectively. Also, $w = 2\mu\theta/\sigma^2$,$b = 2\lambda + \frac{2\mu}{\sigma^2}\bar{\theta}$,$H$ is hypergeometric integrated function as

$$H(w; \lambda; b) = 1 + \frac{1}{b}w + \frac{1}{2!}\frac{\lambda(\lambda+1)}{b(b+1)}w^2 + \frac{1}{3!}\frac{\lambda(\lambda+1)(\lambda+2)}{b(b+1)(b+2)}w^3 + ... \tag{12}$$

where λ_+, λ_- are the solution of the following quadratic function.

$$\frac{1}{2}\sigma^2\lambda(\lambda - 1) + \mu\bar{\theta}\lambda - (r + \delta) = 0, \tag{13}$$

Based on this solution, we can figure that thresholds of new investment (q_l, q_u) are the function of uncertainty (σ^2). To be more precise, the threshold of inactivity range of investment depends on the degree of mean reversion (μ) and uncertainty (σ^2) as appeared in the solution, and depends on the degree of asymmetry of the adjustment costs (c_{21}, c_{22}) as appeared in equation (9). Meanwhile, since this solution cannot be solved analytically, we need to evaluate it numerically. Extensive studies in this field, such as Dixit and Pindyck (1994), have proved that uncertainty could have negative impacts on investment disregarding the parameters. Especially when random variables are perfectly explained by a permanent component of a stochastic process,

[7] See the Appendix for details.

that is $\mu = 0$, Corrado and Holly (2001) argued that the negative effects of uncertainty on investment could become larger. [8]

It should be noted that theoretical analysis with uncertainty and irreversibility about the representative firm does not necessarily imply that the aggregate investment presents apparent discontinuity, although the representative firm has clear discontinuity as Equation (10).

For illustration, we assume that each firm faces the following investment function:

$$I_{it} = \begin{cases} \alpha_{l,it} + \beta q_{it} & if\ q_{it} < q_l \\ 0 & if\ q_l \leq q_{it} < q_u \\ \alpha_{u,it} + \beta q_{it} & if\ q_u \leq q_{it} \end{cases}$$

where each firm is assumed to face stochastic values $q_{it} \sim N(q_t, \sigma_N^2)$, and q_l and q_u are the lower and the upper thresholds of the representative firm, respectively. Then, we can obtain the aggregate investment function as

$$\begin{aligned} \Sigma_{i=1}^{N} I_{it} = F\left(\frac{q^l(\theta_l) - q_t}{\sigma_N^2}\right)(\alpha_{l,it} + \beta q_{it}) + \\ \left(1 - F\left(\frac{q^u(\theta_u) - q_t}{\sigma_N^2}\right)\right)(\alpha_{u,it} + \beta q_{it}) \\ = \left(F\left(\frac{q^l(\theta_l) - q_t}{\sigma_N^2}\right)\alpha_{l,it} + \left(1 - F\left(\frac{q^u(\theta_u) - q_t}{\sigma_N^2}\right)\right)\alpha_{u,it}\right) + \\ \left(F\left(\frac{q^l(\theta_l) - q_t}{\sigma_N^2}\right) + 1 - F\left(\frac{q^u(\theta_u) - q_t}{\sigma_N^2}\right)\right)\beta q_t, \end{aligned} \tag{14}$$

where $F(\cdot)$ denotes the cumulative probability function of the standard normal distribution $N(\mu, \sigma^2)$. From this equation, we can see the aggregate investment function does not necessarily show apparent discontinuity. We should also note several points. First, generally speaking, the aggregate form of nonlinear investment is not necessarily a simple discontinuity form. The form is also affected by the distribution of θ_l, θ_u (or cost adjustment costs, uncertainty and the degree of mean reversion) and profit abilities (q_{it}) of investment. Therefore, the nonlinear function of investment does not seem to be approximated by a simple method such as lagged variables of q_t, since they are affected by several other variables in general. Second, variation in coefficients of Tobin's q_t may be smaller than that in constant term, since coefficients both in the lower and the upper regimes are equal. Third, we rarely observed negative investment as actual firms' behaviors. This may be because desirable

[8] See the Appendix for the solution when $\mu = 0$. The exponent of θ in profit function, which is assumed to be one in this model, can also affect the relationship between investment and uncertainty. Evaluating various solutions under various circumstances is not our research purpose for this paper. See Dixit and Pindyck (1994) for surveys.

negative investment will be fully covered within depreciation costs if there are any. Therefore, for our empirical studies, we usually observe only two regimes; inactive and positive investment.

In the next section, we implement empirical analysis taking these features of irreversible investment model into account.

3 Empirical Studies

As noted in the previous section, parameters of the aggregate investment function could change depending on uncertainty. In this section, we will examine the effects of uncertainty on investment based on investment function of Tobin's q. We have to consider the following features for the estimation as noted in the previous section: uncertainty and investment may relate nonlinearly, investment could be affected both positively and negatively, and several unobserved variables other than uncertainty such as distribution of firms may affect investment parameters. Thus, we will adopt theMarkov Switching Model Approach for our study. In this model, parameters unobserved in the investment function will be set as state variables. Also, the changes of state variables and their probabilities describe uncertainty could have nonlinear effects on investment. [9]

In practice, we adopt the following two-step strategy:

(i) Detecting investment slump and, if any, over-investment by using the usual invest function by Tobin's q and latent variables. When the estimated investment level is below the level of the regular investment function by Tobin's q, the difference is assumed as the degree of investment slump (underinvestment), and vice versa.

(ii) Searching significant factors of uncertainty having significant effects on investment by using detected investment slump and, if any, over-investment as a dependent variable explained by candidates of uncertainty variable in preliminary regression (subsample linear regression);

(iii) Estimating investment equation by using the investment function by Tobin's q with the possible uncertainty variables, and select uncertainty variables that have significant effects on investment.

By this approach, we can efficiently select variables of uncertainty. [10]

[9] Although the state variable is a discrete variable, its probability is continuous. Thus, the expected value of uncertainty effects on investment is described as a smooth transition. If we know all the uncertainty factors and the other factors affecting the effects of those uncertainties on investment, we can use the smooth transition model. However, if we have unknown factors, we should adopt more flexible models such as a Markov switching model. See van Dijk et al. (2002) for details.

[10] We can also extract negative uncertainty effects on investment distinguishing from positive effects. This is the feasible approach of our study to analyze factors of Japan's sluggish.

3.1 Detecting Under- and Over-investment Based on Tobin's q

Before analyzing the effects of uncertainty on investment, we will detect the under-investment period, which is lower than that by the investment function by Tobin's q, and its degree, and vice versa. To be more precise, we will estimate the investment function with three states, using the Markov Switching process with three unobservable state variables (normal state, under state and excess state). In this step, we can test the null hypothesis of no regime change against the hypothesis of regime change with a specific number of states. In this approach, we can also express, for example, underinvestment volume at a certain period as an expected underinvestment, by multiplying estimated parameter of the state by the probability of the state at the period.

As noted in the previous section, constants and coefficients could be nonlinear when investment function by Tobin's q is affected by uncertainty. However, nonlinearity to parameters is also noted to be smaller than that to constants. Thus, in the first step, we adapt the strategy to estimate both models; a model in which both parameters and constants have nonlinearity, and a model in which only constants have nonlinearity. Then, we select the better model based on marginal likelihood estimates. According to our empirical results, the model in which only constants have nonlinearity performed better (showing higher marginal likelihood). We will show the details as follows.

The model is written as

$$
\begin{aligned}
I_t/K_{t-1} = \alpha_{S_t} + \beta_{0.i_q} q_{t-i_q} + \Sigma_{k=1}^{K} \beta_{k,i_k} z_{k,t-i_k} + \epsilon_t \\
S_t = [S_1, S_2, ..., S_M]' \\
Pr[S_t = j, S_{t-1} = i] = p_{ij},\ \Sigma_{j=1}^{M} p_{ij} = 1
\end{aligned}
\tag{15}
$$

where I_t is investment at time t, K_{t-1} is real capital stock at time $t-1$, q_t represents Tobin's q at time t, α and β are parameters, S_t is unobservable state variables at time t, $i.$ indicates the lag order of the variable, p represents probability, and ϵ_t represents error term of variance σ^2. $z_{k,t-i_k}$ mean cash flow or cash-flow-related variables as some previous studies adopted in estimating the investment function. [11] We adopt $CFratio$ (cash flow/nominal capital stock), BS^f (debt-asset ratio), BS^b (bank balance-sheet condition) as candidates of z.

We use quarterly and seasonally adjusted data of the manufacturing industry from the first quarter in 1970 to the third quarter in 2003 (131 periods in total). Details are expressed in the Appendix, and do also estimation method of Tobin's q. [12]

[11] See Hubbard (1996) for an empirical example.

[12] In our model with unobserved variables, external shocks like the oil crises do not affect the analysis of uncertainty. We also analyze uncertainty effects using the sample period between 1975/1Q-2002/3Q, and the results do not affect our main results.

In general, it is not necessary to allow lags because Tobin's q is already derived in consideration of adjustment costs. We, however, consider the possibility that Tobin's q which business leaders recognize in real time could get behind the estimated Tobin's q. Thus, we allow lagged variables of Tobin's q. [13] We set the maximum number of states as three: average state, under-investment state, and over-investment state, the maximum lag length of Tobin's q, and cash-flow-related variables z as three. We also allow no adoption of each cash-flow-related variable. As a result, 1500 combinations ($= 3 \times 4 \times (1+4) \times (1+4) \times (1+4)$) are estimated by using Markov Switching models. For the actual estimation in this study, Markov chain Monte Carlo (MCMC) methods were adopted. [14] [15]

To select an appropriate model, we adopt marginal likelihood, which makes use of sampled data. Based on marginal likelihood, the selected model is reported in Table 1. Table 1 show that the selected number of states is three, the selected lag of Tobin's q is zero, and the selected cash-flow-related variable is only cash flow ratio with lag length two. The signs of both coefficients are significantly positive, as theoretically excepted. Geweke (2001)'s simulation tests, reported in Table 1, show that there are no errors in Monte Carlo simulation. [16] [17]

Marginal likelihood estimation has several methods depending on where it is evaluated among a distribution. For our study, we adopt three different sampling algorithms, that is Chib(1995)'s sampling L_{Chib}, importance sampling L_{IS}, and Bridge sampling L_{BS}. Chib's sampling L_{Chib} is derived at a representative sample's point

[13]The view that Tobin's q differs from one estimated based on sampled data to one business leaders recognize in real time is also presented in Blanchard, Rhee and Summers (1993).

[14]We have confirmed convergence by plotting graphs and by implementing Geweke (1992)'s tests. 12,000 samplings were operated. We exclude the first 2000 samplings and, based on LM tests for serial correlation, pick up five other samplings to avoid correlation among samplings. The same operation is operated for the following MCMC methods in this paper unless it is specifically explained.

[15]We adopt $\alpha_i \sim N(0,2)$, $\beta_{0,.} \sim N(0,2)$,$\gamma_i \sim N(0,2)$, $\sigma^2 \sim IG(4,4)$ $q_i \sim Dirichlet(e_{0,i1},...,e_{0,iK})$, in which $e_{0,ii} = K$ and $e_{0,ij} = 1 \; if \; i \neq j$, as prior distributions. These priors are relatively vague. Although we try other several prior distributions, those changes do not affect our main results.

[16]The model in which the coefficient of Tobin's q also has nonlinearity is written as

$$I_t/K_{t-1} = \alpha_{S_t} + \beta_{0.i_q,S_t} q_{t-i_q} + \Sigma_{k=1}^{K} \beta_{k,i_k} z_{k,t-i_k} + \epsilon_t$$

$$S_t = [S_1, S_2, ..., S_M]'$$

$$Pr[S_t = j, S_{t-1} = i] = p_{ij}, \; \Sigma_{j=1}^{M} p_{ij} = 1$$

By estimating 1500 combinations models and marginal likelihood as is the case with no linearity in the coefficient, we find that the marginal likelihood estimates of this case are significantly lower than that of maximal estimates of the case of the fixed coefficient of Tobin's q, which is not inconsistent with our theoretical analysis in the previous section.

[17]This is an unrestricted estimation. Marginal likelihood indicates plausibility of a model, that is determined as $\pi(m|D)$. This is derived by integrating conditional posterior distribution $\pi(\theta, m|D)$ with respect to parameter θ and by canceling it out. This is also referred as "model likelihood."

(such as mean or mode). Importance sampling L_{IS} is derived from samples from the density function (posterior density) in the denominator of the marginal likelihood function [18]. Bridge sampling L_{BS} is derived by optimal weighting of samplings from both functions on the denominators and numerators of the marginal likelihood function. [19]

Chart 1 shows data used in the selected nonlinear model, and Chart 2 shows actual data, estimated value, and estimated value in normal state. In Chart 2, when a value is below that in normal state, it is underinvestment. Also, it is overinvestment when the value is above that in normal state. To see transitions in each state clearly, Chart 3 shows a stochastic sequence of probabilities of each state.

In Chart 3, excluding the so-called bubble economy term in the late 1980s and the beginning of 1990s where the probability of state in overinvestment is remarkably high, probability of modest- and under- investment appear alternately. Especially, probability of underinvestment appears highly in the 1980s, after the first oil shock, and in the 1990s, after the collapse of bubble economy.

3.2 Extracting Uncertainty

In this section, we will extract uncertainty out of economic variables that affects to underinvestment. In evaluating the effects of uncertainty on investment, we adopt two strategies.

(i) Extracting uncertainty not from profits or stock prices, which are supposed to contain all uncertainty firms are facing, but from several exogenous fundamental factors of uncertainty such as exchange rates. By distinguishing factors of uncertainty, we can verify which factor is more influential. In addition, since profits and stock prices are, at least to some extent, endogenous variables for firms, it may fail to extract uncertainty properly. That is because, for example, firms will make adjustments of capital and labor inputs to prevent a situation in which uncertainty of profits is affected by an increase of fundamental factors of uncertainty. This problem can be avoided by adopting exogenous fundamental factors of uncertainty. [20]

(ii) Separating uncertainty of each economic variable into permanent and transitory components. This is because the effects of uncertainty on investment could differ based on permanent and transitory uncertainty as noted previously.

[18] See the Appendix for details.

[19] It is recommended to examine by importance sampling or Bridge sampling when a model may be not necessarily single modal as unrestricted Markov Switching models. See also Geweke and Keane (2001), Chib (2001) for details on marginal likelihood.

[20] We also analyze the effects of uncertainty on profits and stock prices. See the Appendix for analyses on uncertainty of profits and stock prices.

To extract permanent and transitory components, we adopt the Kim and Nelson (1999) method. Thus, we assume the following model:

$$y_t = z_t + x_t, \tag{16}$$

$$z_t = z_{t-1} + e_t, e_t \sim N(0, \sigma^2_{e,t}), \tag{17}$$

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + u_t, u_t \sim N(0, \sigma^2_{u,t}) \tag{18}$$

where y_t is the log of economic variables, and e_t and u_t are error terms independent of each other. We assumed that the variances of the shocks to the two components are subject to endogenous regime shifts, which lead to multi-state Markov-switching variances:

$$\sigma^2_{e,t} = \Sigma^M_{j=1} \sigma^2_{e,j} S_{1,j_t}, \tag{19}$$

$$\sigma^2_{u,t} = \Sigma^{M'}_{j=1} \sigma^2_{u,j} S_{2,j_t}, \tag{20}$$

where

$$S_{1,mt} = 1 \ if \ S_{1t} = m; S_{1,mt} = 0 \ otherwise, \ m = 1, 2, ..., M, \tag{21}$$

$$S_{2,m't} = 1 \ if \ S_{2t} = m'; S_{2,m't} = 0 \ otherwise, \ m' = 1, 2, ..., M', \tag{22}$$

and S_{1t} and S_{2t} are two independent first-order Markov-switching variables with the following transition probabilities:

$$p_{1,ij} = Pr[S_{1t} = j | S_{1,t-1} = i], \Sigma^M_{j=1} p_{1,ij} = 1,$$

$$p_{2,ij} = Pr[S_{2t} = j | S_{2,t-1} = i], \Sigma^{M'}_{j=1} p_{2,ij} = 1,$$

In our study, we arbitrary adopt three as the number of states, that is $M = 3$. Due to this adoption, we can simply describe variance as large, middle, and small distribution. Nine variables, such as real sales, output prices, input prices, import prices, investment-goods prices, number of bankruptcies, liabilities to sales ratio of bankruptcies, long-term interest rates, and exchange rates, are adopted to extract uncertainty. Estimation period is the same as that for investment function unless we have the problem of data availability. For variables on which quarterly data are available, it is estimated with seasonally adjusted quarterly data. And if monthly data is available, it is estimated with seasonally adjusted monthly data; estimated monthly data is converted into quarterly aggregation. For the actual estimation, MCMC methods are adopted because it contains both state variables following the Markov switching stochastic process, and latent variables containing transitory and permanent components.

To comprehend multiple uncertainties, we construct five indices for each variable:

(i) probability of the middle and large variance states in the permanent component;

(ii) probability of the large variance state in the permanent component;

(iii) probability of the middle and large variance states in the transitory component;

(iv) probability of the large variance state in the transitory component;

(v) ratio of the permanent component change to the transitory component change.

Extracted permanent component and transitory component are shown in the Appendix Charts.

3.3 Screening Uncertainty Variables Affecting Investment

Models using extracted 45 (=9 variables $\times$ 5 indices) uncertainty variables have approximately 35 trillion ($2^{45} = 35,184,372,088,832$) combinations. [21]

However, it is not a realistic approach to repeat the Monte Carlo exercise for 35 trillion times. Therefore, in this step, we screen candidate uncertainty variables by testing their significance in subsampling linear regression models in which estimated deviations from average state of investment are dependent variables. Although uncertainty could have nonlinear effects on investment as noted previously, we can screen candidate uncertainty based on the significance of subsample linear models approximately. For the actual estimation, the following strategies are adopted, where sample size of full sample is defined as T.

(0) Starting period of subsample for a simple linear regression is $i = 1$.

(1) We will implement 45 simple regressions for subsets with starting period i and sample size n.

(2) After implementing simple linear regressions, we pick up best $\min\{n-2, 8\}$ candidate uncertainty variables which show significant negative effects on investment based on information criterion.

(3) For all combination ($2^{\min\{n-2,8\}}$) of selected uncertainty variables, we implement multivariate linear regression and select the best combinations that show significant negative effects on investment and best Bayesian Information Criterion. For the best fitted regression, we calculate the contribution ratio for each independent variable and store the ratio as data for starting period i.

(4) Shift the starting period by one, and repeat the above procedures until the starting period comes to

[21] We transformed each uncertainty variable into deviation from the mean of each uncertainty variable.

$T - n + 1$.

(5) Aggregate contribution ratios calculated $(T - n + 1)$ times by each variable. We suppose higher order variables as candidates of uncertainty indices for nonlinear investment functions. [22] The size of subsample was selected as 14 periods based on the aggregate contribution ratio that best describes a model among estimations with an estimation period from 5 to 131 (= full sample). The reason why the linear regression is better fitted with subsample estimation rather than full sample may be that the effects of uncertainty on investment have varied over time.

We implement the above procedure using three kinds of dependent variables: estimated under-investment, estimated over-investment, estimated deviation from average investment (under-investment plus over-investment). As for the case with estimated over-investment dependent variable, we cannot obtain any significant candidate. [23] When both cases in which dependent variables are "under-investment" and "deviation from average (under-estimated plus over-estimated)" are estimated, we can obtain the same eleven significant candidates in the same order, which have significant negative effects on investment. Then, we pick up the upper seven variables as candidates out of 11 variables which have significantly negative effects on investment based on preliminary subsampling simple linear regression. That is: import price in the permanent component (large); exchange rate in the permanent component (large); exchange rate in the transitory component (large and middle); exchange rate of the ratio of the permanent component change to the transitory component change; import price in the permanent component (large and middle); liabilities (to sales ratio) of bankruptcies of the ratio of the permanent component change to the transitory component change; and number of bankruptcies of the ratio of the permanent component change to the transitory component change. [24]

[22] In this step, uncertainty at period t is assumed to affect investment at period t. Although we have also implemented the same model specification search with lagged uncertainty variables, only current variables (without any lag) are selected.

[23] This is a subject for future study.

[24] Eleven variables that have significant negative effects on investment based on preliminary subsampling simple linear regressions are (in descending order): "import price in the permanent component (large)," "exchange rate in the permanent component (large)," "exchange rate in the transitory component (large and middle)," "exchange rate of the ratio of the permanent component change to the transitory component change," "import price in the permanent component (large and middle)," "liabilities (to sales ratio) of bankruptcies of the ratio of the permanent component change to the transitory component change," "number of bankruptcies of the ratio of the permanent component change to the transitory component change," "exchange rate in the permanent component (large and middle)," "import price of the ratio of the permanent component change to the transitory component change," "investment-goods prices in the permanent component (large)," and "investment-goods prices of the ratio of the permanent component change to the transitory component change." Although we have estimated nonlinear investment functions with omitted lower variables from our main study, we could not have fine performances, based on marginal likelihood.

3.4 Investment Function Estimation including Components that affect Investment

By using the selected candidate variables of uncertainty, we estimate investment function. We adopt a following model which allow nonlinear parameters of uncertainty component in consideration of the possibility that effects of uncertainty on investment can be nonlinear, that is,

$$I_t/K_{t-1} = \alpha_{S_t} + \beta_{0,0}q_t + \beta_{1,2}z_{1,2} + \Sigma_{j=1}^{m}\gamma_{j,S_{\gamma t}}z_{j,t} + \epsilon_t, \tag{23}$$

$$S_t = [S_1, S_2, S_3]'$$

$$Pr[S_t = l, S_{t-1} = k] = p_{kl},\ \Sigma_{l=1}^{3}p_{kl} = 1$$

$$S_{\gamma t} = [S_1, S_2]'$$

$$Pr[S_t = l, S_{t-1} = k] = p_{kl},\ \Sigma_{l=1}^{2}p_{kl} = 1$$

where $z_t = (z_{1,t}, z_{2t}, ..., z_{mt})'$ is the uncertainty component, and $\gamma_{jS_{\gamma t}}$ is the parameter. This model describes nonlinear and time-varying effects of uncertainty on investment. [25] We implemented all combinations with seven candidates and, based on marginal likelihood, we selected the model with three uncertainty variables, that is "exchange rate in the transitory component (large and middle)," "exchange rate in the permanent component (large and middle)," and "liabilities (to sales ratio) of bankruptcies of the ratio of the permanent component change to the transitory component change." Estimated results are shown in Table 2. Coefficients of Tobin's q and cash-flow-related variable are significantly positive, as theoretically expected. Smaller coefficient of each selected uncertainty component variable is significantly negative, which mean that uncertainty has negative effects on investment.

Comparing three types of marginal likelihood estimates, marginal likelihood estimates of this model in Table 2 are significantly greater than those of the model without the uncertainty components in Table 1. Geweke (2001)'s simulation tests reported no error in the Monte Carlo simulation.

From this empirical study using Japanese data, it is verified that uncertainty has significantly negative effects on investment. [26]

[25] We have estimated the functions in which each uncertain variable $z_{j,t}$ have each state variable $S_{j,\gamma t}$. However, we could not have better perfotmance, based on marginal likelihood.

[26] We also estimate marginal likelihood with fixed uncertainty parameter, which suggests smaller marginal likelihood. This result indicates that parameters of uncertainty variables are preferred as a time-varying parameter of our model.

As we suppose change of the contribution of uncertainty component variables z_{it} at time t as $Cont_{it}$, we can write,

$$Cont_{it} = (\Sigma_{j=1}^{2}(Prob_{ijt} \times \gamma_{ij})) \times (z_{it}) \tag{24}$$

where $Prob_{ijt}$ represents state probability of i-th uncertainty component of j-the state at time t. Each z is transformed into deviation from its average, $Cont_{it}$ means the change of the negative contribution of the z on investment.

Chart 4 shows the actual values, the estimated values, the estimate values excluding uncertainty component effects, and the estimated average values by the selected model with uncertainty variables. In Chart 4, the difference between the estimated values and the estimated values excluding uncertainty component contributions seems relatively small compared to the deviation from investment which is explained in unobserved variables. Effects of this specified uncertainty on investment in our study, however, do not necessarily have a small impact. [27] First, the differences among constant papameters, $\alpha_1, \alpha_2, \alpha_3$, become smaller in Table 2 than those in Table 1, which may be due to adopting the uncertainty variables. Second, Chart 5 shows the stochastic sequence of probability of each state as Chart 3. Due to contributions of the uncertainty variables, the probability of under-investment seems frequently smaller. Besides, Chart 6 shows the changes of the contributions of the uncertainty component estimated the selected model which includes uncertainty variables. Chart 6 implies that the negative effects of selected uncertainty variable on investment ratio $((I/K)$ are not necessarily moderate, with an impact is about 0.2 percent on the change of I/K on average (approximately 0.5 percent for maximum), and about 3 percent on the change of real investment (approximately 9 percent for maximum). As effects by component, exchange rate in the transitory component has negative effects on investment through the sample period while the permanent component has similar significant effects up until the 1980s, which may reflect the period of the rapid yen appreciation. [28] Uncertainty of bankruptcy, such as liabilities of bankruptcies (to sales ratio) variable, have larger negative impacts on investment as the ratio of the permanent component change increased in the late 1990s. [29]

[27] We also estimated the models in which S_t have one state and two states. However, we could not obtain better performances than that of the selected model with the three states. The deviation from investment, wich is explained in unobserved variables, may reflect firms' ratinalization for structural changes, effects of financial crises, and so on. It is a subject for future study.

[28] Precisely speaking, in Chart 6, positive contributions refer not to positive effects on investment, but smaller negative effects on investment.

[29] It is more appropriate to estimate a model that includes an investment function and a model to separate uncertainty variables into permanent components and transitory components. It is a subject for future study.

4 Implications

In this paper, we have examined the effects of uncertainty on investment by a nonlinear and time-varying model using Japanese investment data. According to our empirical results, uncertainty has significant negative effects on investment. Also, it is noted that exchange rate in the transitory component has negative effects throughout the estimation period, and the exchange rate in the permanent component particularly has negative effects up until the 1980s. Uncertainly of bankruptcy has resulted to negatively affect investment, especially in the last half of the 1990s.

These results could give some hints to understand the recent Japanese economic slump, because our study has clarified an additional aspect that causes a sluggish Japanese economy other than the well-recognized zero bound of nominal interest rate and delay of the structural changes, which is uncertainty repressing investment. To expect a sustainable economic recovery, it will be key to lower uncertainty. Since the exchange rate shift turned out to have negative effects on investment as verified by our research, it implies that the importance of economic policy to avoid unwanted wild swing of exchanges. Also, since uncertainty of bankruptcy has had to have larger effects on investment recently, we would add importance of the economic policy to minimize downward pressure on business sentiment by cutting off the bankruptcy chain.

A Deriving Solution of Bellman Equation

According to Abel and Eberly (1997), we derive the solution. Assume a linear capital stock function as,

$$V(K,\theta) = q(\theta)K + G(\theta)$$

where $q(\theta)$ is the shadow price of investment, and $G(\theta)$ is a constant. By substituting the above function into a Bellman equation, the differential equation which supposed to be differentiable by all K is rewritten as

$$rq = A\theta - \delta q + \mu(\bar{\theta} - \theta)\theta q_\theta + \frac{1}{2}\sigma^2\theta^2 q_{\theta\theta} \; . \tag{25}$$

In general, the solution of this differential equation is given

$$q(\theta) = C\theta^\lambda h(\theta) \; . \tag{26}$$

This is substituted into equation (25), and when it suppose to be solvable for all θ, we have

$$\frac{1}{2}\sigma^2\theta h_{\theta\theta}(\theta) + h_\theta(\lambda\sigma^2 + \mu\bar{\theta} - \mu\theta) - \mu\lambda h(\theta) = 0 \tag{27}$$

which is Kummer's differential equation. Thus, we can solve $q^I(\theta),\ I = l, u$ in Kummer's differential equation.

$$q^u(\theta) = \frac{1}{r+\delta-\mu\bar{\theta}}A\theta + B_{\_}\theta^{\lambda_-} H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_-; b\right)\ \ if\ \theta \in [\theta_u, \infty), \tag{28}$$

$$q^l(\theta) = B_+\theta^{\lambda_+} H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_+; b\right)\ \ if\ \theta \in [0, \theta_l], \tag{29}$$

As for this function, we assume $w = 2\mu\theta/\sigma^2,\ b = 2\lambda + \frac{2\mu}{\sigma^2}\bar{\theta},\ H$ as following hyper geometric integrated function.

$$H(w;\lambda;b) = 1 + \frac{1}{b}w + \frac{1}{2!}\frac{\lambda(\lambda+1)}{b(b+1)}w^2 + \frac{1}{3!}\frac{\lambda(\lambda+1)(\lambda+2)}{b(b+1)(b+2)}w^3 + ... \tag{30}$$

λ is supposed to be the solution of the following quadratic equation,

$$\frac{1}{2}\sigma^2\lambda(\lambda-1) + \mu\bar{\theta}\lambda - (r+\delta) = 0, \tag{31}$$

where q_l supposed to have a positive solution and q_u a negative solution, due to a boundary condition $q(0) = 0$, $q(\infty) = 0$.

Consequently, we have now figured that thresholds θ_u and θ_l is a function with variance σ^2. Under the conditions adopted in this paper, thresholds (θ_u, θ_l) and parameters $(B_{\_}, B_+)$ are not solvable analytically; it is necessary to evaluate numerically. [30]

When $\mu = 0$, that is when shifts are all the permanent component, it is given

$$q^u(\theta) = \frac{1}{r+\delta-\mu\bar{\theta}}A\theta + B_{\_}^I\theta^{\lambda_-} \qquad for\ [\theta_u, \infty), \tag{32}$$

$$q^l(\theta) = B_+^E\theta^{\lambda_+} \qquad for\ [0, \theta_l). \tag{33}$$

Assuming the smooth pasting conditions additionally, we have

$$\theta_u = \frac{r+\delta}{A}\left[c_{21}\left(\frac{1-\lambda_+ + \lambda_-}{1-\lambda_-}\right)\left(\frac{\theta^u}{\theta_l}\right)^{\lambda_-}\right], \tag{34}$$

$$\theta_l = \frac{r+\delta}{A}\left[c_{22}\frac{\lambda_+ - \lambda_-}{1-\lambda_-}.\right] \tag{35}$$

[30] In this paper, a numerical approach has not been attempted. See Dixit and Pindyck (1994) for details about the numerical approach.

B Data

Data was prepared as follows:

Real Stock of Capital Financial Statements Statistics of Corporations by Industry (Quarterly), manufacturing (with capital of 1 billion yen or more), Tangible capital stock is calculated by using amount of increase in other fixed assets by the permanent inventory method. Discontinuity adjusted. Tangible assets depletion rate is premised on a computation in Ogawa and Kitasaka (1998). Real term is calculated by using investment-goods prices.

Investment-goods Prices calculated by using matrix of capital from Interindustry-Relations Table and wholesale price index by commodities.

Real Investment Financial Statements Statistics of Corporations by Industry (Quarterly), manufacturing (with capital of 1 billion yen or more), calculated by using amount of increase in other fixed assets and investment-goods prices, discontinuity adjusted.

Cost of Liabilities (interest payments + bond interest payment)/(long- and short-term borrowings + bonds + bills discounted)

Marginal Return of Capital (operating revenue (before depreciation cost subtracted)/real stock of capital at previous end of period), by following Blanchard, Rhee and Summers (1993).

proxy variable of marginal q (marginal return of capital/capital cost)/(price index of investment goods). Due to mutual share holding and the bubble economy, capital cost is calculated by using only cost of liabilities according to Suzuki (2002). Suzuki (2001) indicates that the effects excluding cost of equity are not so large.

Cash Flow Ratio Cash flow ratio= $(cash\ flow)_t$ / $(price\ of\ investment\ goods \times realcapitalstock)_{t-1}$

Debt-asset Ratio Debt/ (Market-valued assets)

Capital Ratio of Banks (Shareholders' equity+ Capital gains-losses from securities +Loan-loss provisioning-Risk management assets (if available)-Deferred tax assets (if available))/Assets

C Marginal Likelihood Estimation

C.1 Marginal likelihood estimators

Suppose that $y^N = (y_1, y_2, ..., y_N)$ is a sequence of N observations and that y takes either discrete or continuous values. We assume that the distribution $f(y_1, ..., y_N | I^N, \theta)$ of y^N depends on a sequence $I^N = (I_1, ..., I_N)$ of latent (unobservable) variables I_t taking values in a discrete space $\{1, ..., K\}$ as well as on a model parameter θ. The distribution $\pi(I^N|\eta)$ of the latent variables I^N has to be characterized up to a hyperparameter η.

Model likelihood is defined by

$$L(y_N) = \int f(y_1, y_2, ..., y_N|\phi)\pi(\phi)\nu(d\phi), \tag{36}$$

where ϕ is some unknown parameter vector, explicit formula for the likelihood $f(y_1, y_2, ..., y_N)$ is available, and ν is a legitimate measure around $\sigma-$algebra. From Equation (36), the model likelihood is equal to the normalized constant of the posterior density $\pi(\phi|y^N) \propto \pi^*(\phi|y^N) = f(y^N|\phi)\pi(\phi)$. Let $q(\phi)$ be a density with known normalized constant, which is some simple approximation to the posterior $\pi(\phi|y^N)$.

Importance sampling (Fruhwirth-Schnatter (1995)) is defined as

$$\hat{L}_{IS}(y^N) = \frac{1}{L}\frac{\pi^*(\tilde{\phi}^{(l)}|y^N)}{q(\tilde{\phi}^{(l)})} \tag{37}$$

with $\tilde{\phi}^{(1)}, ..., \tilde{\phi}^{(L)}$ being an $i.i.d.$ sample from $q(\theta)$. This is the marginal likelihood evaluation method by using the sampled value derived from functions in the numerator.

Meanwhile, there could be a method evaluating by sampled value derived from functions in the numerator, that is reciprocal importance sampling estimator (Gelfand and Dey (1994)) defined as

$$\hat{L}_{RI}(y^N) \left\{ M^{-1}\Sigma_{m=1}^{M} \frac{q(\phi^{(m)})}{\pi^*(\phi^{(m)}|y^N)} \right\}^{-1} \tag{38}$$

where $\phi^{(m)}, m = 1, ..., M$ is an $i.i.d.$ sample from $\pi(\phi|y^N)$.

There are also integrated estimation methods.

Let $\alpha(\phi)$ be an arbitrary function such that $\int \alpha(\phi)\pi(\phi|y^N)q(\phi)\nu(d\phi) > 0$. Bridge sampling (Meng and Wong (1996)) is based on the following identity:

$$1 = \frac{\int \alpha(\phi)\pi(\phi|y^N)q(\phi)\nu(d\phi)}{\int \alpha(\phi)q(\phi)\pi(\phi|y^N)\nu(d\phi)} = \frac{\int \alpha(\phi)\pi^*(\phi|y^N)q(\phi)\nu(d\phi)}{L(y^N)\int \alpha(\phi)q(\phi)\pi(\phi|y^N)\nu(d\phi)} \tag{39}$$

which yields:

$$L(y^N) = \frac{E_q(\alpha(\phi)\pi^*(\phi|y^N)))}{E_\pi(\alpha(\phi)q(\phi))} \tag{40}$$

where E_f is the expectation with respect to a density $f(\cdot)$. By using $\phi^{(m)}$, $\tilde{\phi}^{(l)}$ and $q(\phi)$, the bridge sampling estimator is defined as

$$\hat{L}_{BS}(y^N) = \frac{\hat{E}_q}{\hat{E}_\pi} = \frac{L^{-1}\Sigma_{l=1}^{L}\alpha(\tilde{\phi}^{(l)})\pi^*(\tilde{\phi}^{(l)}|y^N)}{M^{-1}\Sigma_{m=1}^{M}\alpha(\phi^{(m)})q(\phi^{(m)})} \tag{41}$$

In the matter of selection of weight α, Meng and Wong (1996) discussed an asymptotically optimal choice of $\alpha(\phi)_{opt}$, which minimizes the expected relative mean squared error of the estimator $\hat{L}_{BS}(y^N)$ for *i.i.d.* drew from $\pi(\phi|y^N)$ and $q(\phi)$:

$$\alpha(\phi)_{opt} \propto \frac{1}{Lq(\phi) + M\pi(\phi|y^N)}. \tag{42}$$

We call the corresponding sampling estimator $\alpha(\phi)_{opt}$ the optimal bridge sampling estimator $\hat{L}_{BS_{opt}}(y^N)$.

In practice, $\alpha(\phi)_{opt}$ for marginal likelihood estimation itself also depends upon the unknown marginal likelihood. Thus, following Meng and Wong (1996), we can derive $\hat{L}_{BS_{opt}}(y^N)$ by the following iterative procedure:

$$\hat{L}^{(t)}_{BS_{opt}} = \hat{L}^{(t-1)}_{BS_{opt}} \frac{L^{-1}\Sigma_{l=1}^{L} \frac{\hat{\pi}(\tilde{\phi}^{(l)}|y^N)}{Lq(\tilde{\phi}^{(l)})+M\hat{\pi}(\tilde{\phi}^{(l)}|y^N)}}{M^{-1}\Sigma_{m=1}^{M} \frac{q(\phi^{(m)})}{Lq(\phi^{(m)})+M\hat{\pi}(\phi^{(m)}|y^N)}} \tag{43}$$

In our studies, we adopt the importance sampling marginal likelihood estimator $\hat{L}_{IS}(y^N)$ as starting value $\hat{L}^{(0)}_{BS_{opt}}(y^N)$.

Another marginal likelihood estimator derived by Chib(1995) is defined as

$$\hat{L}_{CH}(y^N) = \frac{f(y^N|\theta^*, \eta^*)\pi(\theta^*, \eta^*)}{\hat{\pi}(\theta^*, \eta^*|y^N)} \tag{44}$$

where (θ^*, η^*) and $\pi(\theta^*, \eta^*|y^N)$ are estimated from MCMC output of the Gibbs sampler.

The performance of these estimators can be measured in terms of the expected relative mean-square error (RE^2).

$$RE^2(\hat{L}(y^N)) = \frac{E(\hat{L}(y^N) - L(y^N))^2}{L^2(y^N)} \tag{45}$$

Re^2 is also an approximation to the expected absolute mean-square error of $log\hat{L}(y^N)$.

$$E(log\hat{L}(y^N) - logL(y^N))^2 \approx RE^2(\hat{L}(y^N)) \tag{46}$$

C.2 Selection of the Importance Density $q(\phi)$

To implement the marginal likelihood estimations, we have to select an importance density $q(\phi) = q(\theta, \eta)$. In this paper, we adopt the procedure proposed by Chib (1995) and Fruhwirth-Schnatter (1995) for switching models, in which densities can be multimodal. Let Ω denote the support of parameter $\psi = (I^N, \theta, \eta)$, ν denote legitimate measure of Ω around $\sigma-$algebra, and $K(\psi|\psi')$ denote the density of the transition kernel of the MCMC sampler with respect to ν.

$$\pi(\psi|y^N) = \int_\Omega K(\psi|\psi')\pi(\psi'|y^N)\nu(d\psi') \tag{47}$$

In the case of the Gibbs sampling, the density $K(\phi|phi')$ takes the following form:

$$K(\psi|\psi') = K_\theta(\theta|I^N, y^N, \theta')\pi(\eta|I^N)\pi(\theta,\eta)', y^N), \tag{48}$$

By splitting θ into D blocks $(\theta_1, ..., \theta_D)$, K_θ can be expressed as,

$$K_\theta(\theta|I^N, y^N, \theta') = \Pi_{d=1}^D \pi(\theta_d|\theta'_{j<d}, \theta_{j>d}, I^N, y^N). \tag{49}$$

The importance density is constructed as a mixture density derived from the MCMC draws,$\phi^{(1)}, ..., \phi^{(s)}$ by Rao-Blackwellization (Gelfand and Smith (1998)).

$$q(\theta,\eta) = \frac{1}{S}\Sigma_{n=1}^S \pi(\eta|(I^N)^{(n)})\Pi_{d=1}^D \pi(\theta_d|\theta_{j<d}^{(n)}, \theta_{j>d}^{(n-1)}, (I^N)^{(n)}, y) \tag{50}$$

D Using Profit or Stock Price Variables as Uncertainty

Since changes of corporate profits and stock prices are reflected to uncertainties in the business environment, we can extract the uncertainty from these variables and use them to analyze the effects of uncertainty on investment. The possible variables for uncertainty extraction are operating profits (including depreciation cost) and stock prices (Topix). As the same method implemented in the main body, five types of uncertainty indices are extracted for each by Kim and Nelson (1999). We specify candidate uncertainty indices by testing their significance in subsampling linear regression models. As the result, three variables turned out to have significant impact on investment. Those were "operating profit in the permanent component (large)," "operating profit of the ratio of the permanent component change to the transitory component change," and "stock prices of the ratio of the permanent component change to the transitory component change." [31] Investment function is estimated with all three uncertainty variables, and the results are not improved compared to the model without the uncertainty. In short, although extracted uncertainty indices have significant negative impact on investment in subsampling linear regression, they do not perform well in estimation throughout the full sample

[31] The size of sample is set the same as 14 periods.

period. That may be because; (i) those variables contained too much information on uncertainty and may include noizy information that is not useful for firms' investment plans, and (ii) extracted uncertainty indices themselves are affected by their corporate activities.

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Table 1: Selected Nonlinear Model: Without Uncertainty Variables

parameters	estimates	s.e.
p_{11}	0.913	0.035
p_{12}	0.059	0.029
p_{21}	0.162	0.060
p_{22}	0.749	0.075
p_{31}	0.231	0.116
p_{32}	0.324	0.115
α_1	0.053	0.004
α_2	0.068	0.005
α_3	0.083	0.006
$\beta_{0,0}$	0.004	0.002
$\beta_{1,2}$	0.209	0.041
σ^2	2.632e-005	4.375e-006
Marginal Likelihood		
	estimates	s.e.
L_{IS}	449.758	0.288
L_{BS}	438.711	0.851
L_{Chib}	444.063	0.700
Geweke(2001)'s simulation comparison test		
The Number of significant differences at the 5 % significant level		
0 across 90 momments		

Note 1: L_{IS}, L_{BS}, L_{Chib}mean marginal likelihood by importance sampling approach, Bridge sampling approach, and Chib (1995)'s approach, respectively.
Note 2: The nonlinear case in which both the constant term and the coefficient of Tobin's q are time-varying parameters.
$\max_\Omega L_{IS} < 449.758$
$\max_\Omega L_{BS} < 441.824$
$\max_\Omega L_{Chib} < 438.711$
where Ω is a set of specifications to be searched.
Note 3: We simulate 150,000 iterations across the first and second moments of the prior parameters. See Section 3 for prior parameter setting.

Table 2: Selected Nonlinear Model: With Uncertainty Variables

parameters	estimates	s.e.
p_{11}	0.849	0.050
p_{12}	0.114	0.045
p_{21}	0.222	0.069
p_{22}	0.664	0.097
p_{31}	0.202	0.089
p_{32}	0.351	0.104
α_1	0.032	0.010
α_2	0.044	0.011
α_3	0.058	0.013
$\beta_{0.0}$	0.014	0.004
$\beta_{1,2}$	0.287	0.064
$p_{\gamma,11}$	0.515	0.167
$p_{\gamma,21}$	0.520	0.188
γ_{11}	-0.158	0.120
γ_{12}	-0.296	0.148
γ_{21}	-0.010	0.007
γ_{22}	-0.021	0.012
γ_{31}	-0.004	0.003
γ_{32}	-0.010	0.005
σ^2	2.254e-005	4.226e-006
Marginal Likelihood		
	estimates	s.e.
L_{IS}	455.649	0.221
L_{BS}	446.323	0.426
L_{Chib}	451.315	0.764
Geweke(2001)'s simulation comparison test		
The Number of significant differences at the 5 % significant level		
0 across 230 momments		

Note 1: L_{IS}, L_{BS}, L_{Chib}mean marginal likelihood by importance sampling approach, bridge sampling approach, and Chib (1995)'s approach, respectively.
Note 2: We simulate 150,000 iterations across the first and second moments of the prior parameters. See Section 3 for prior parameter setting.

Chart 1a. Investment

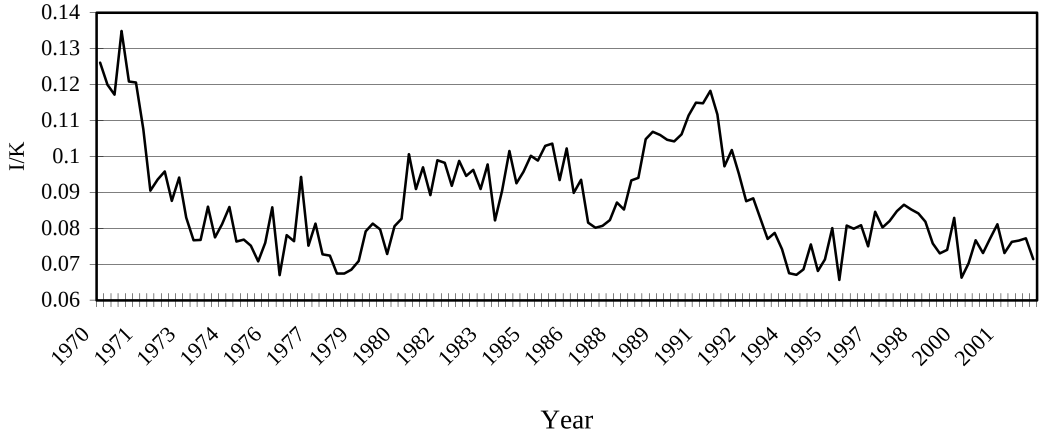


Chart 1b.Tobin's q

Chart 1c .Liquidity Ratio

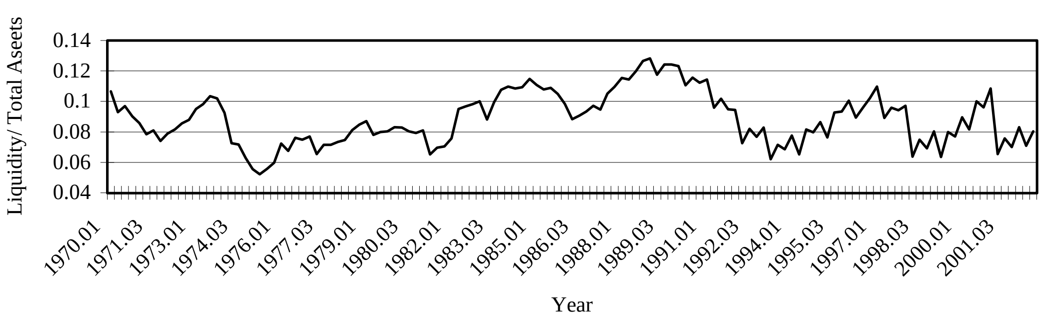


Chart 2. Estimate Results of Selected Model:
Without Uncertaity Variables

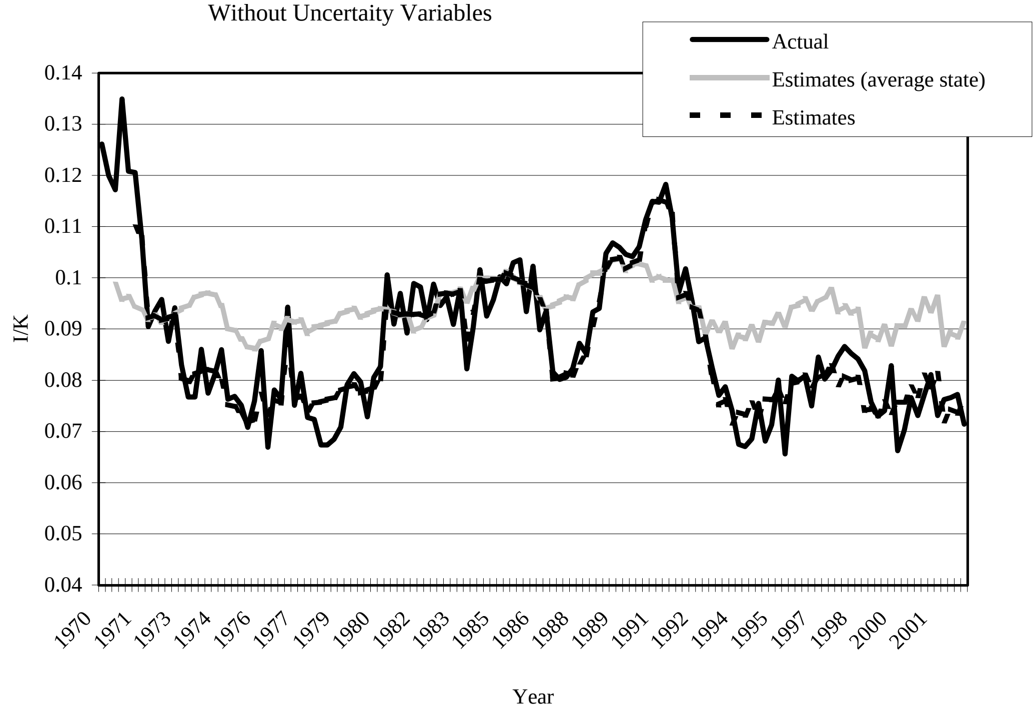


Chart 3. Probabilities of Each State:
Selected Model (without uncertainty variables)

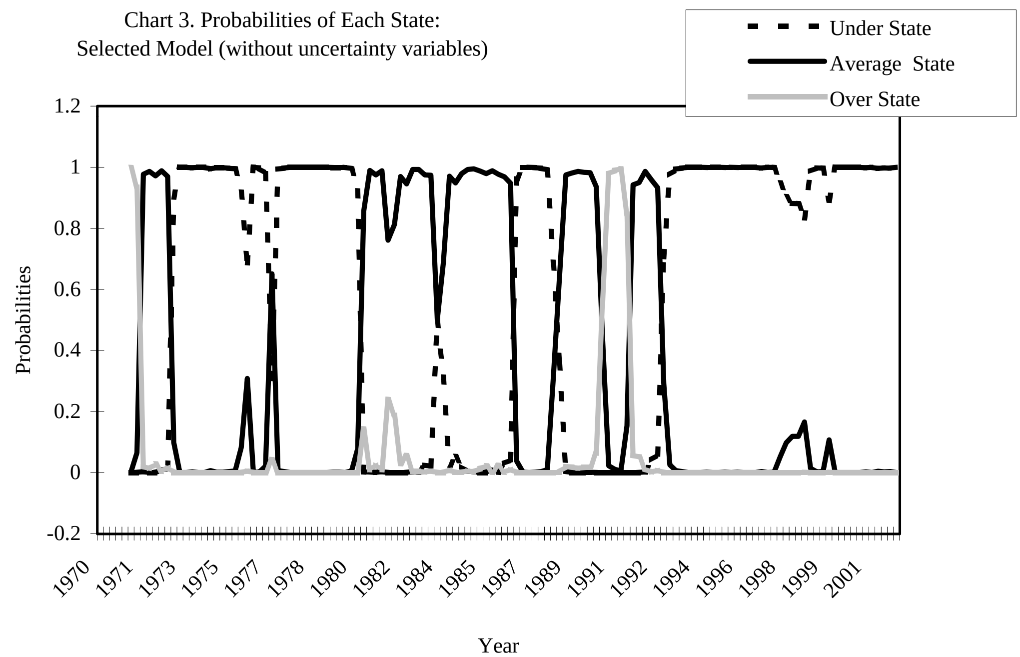


Chart 4. Estimate Results of Selected Model:
With Uncertainty Variables

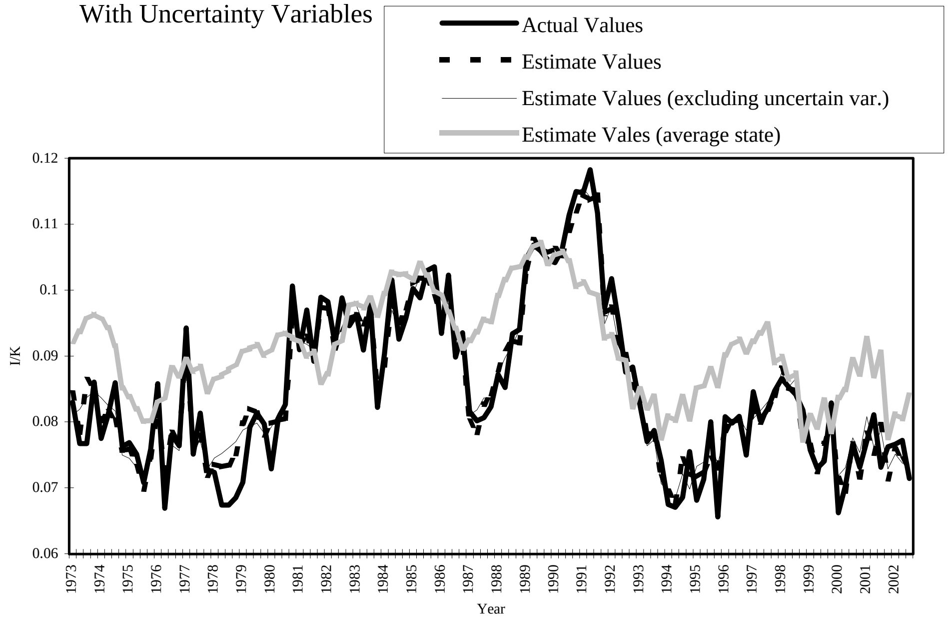


Chart 5. Probabilities of Three States:
Selected Model with Uncertainty Variables

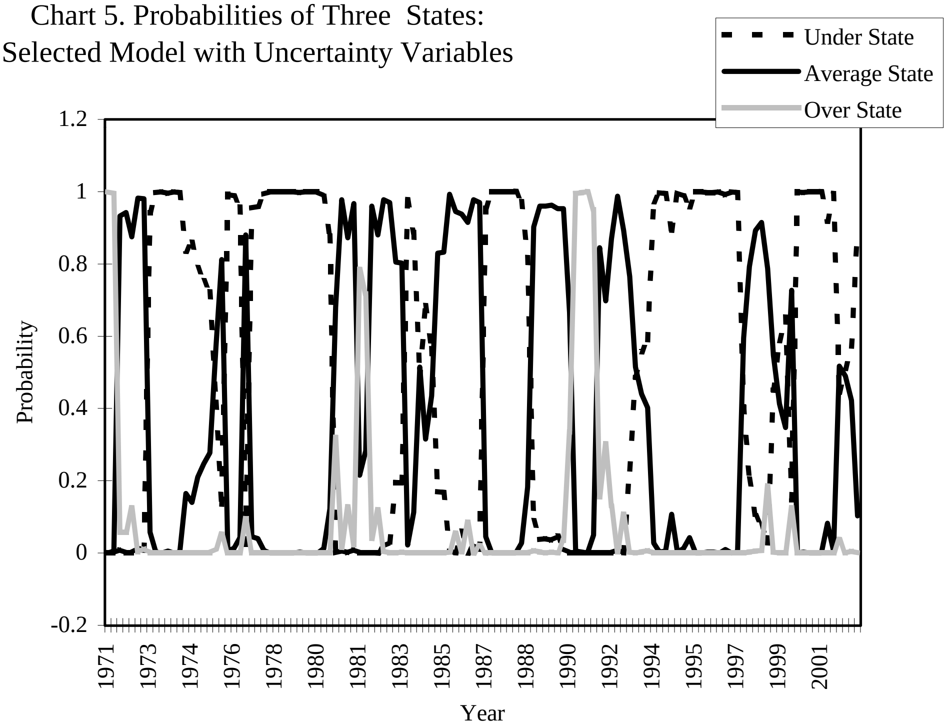
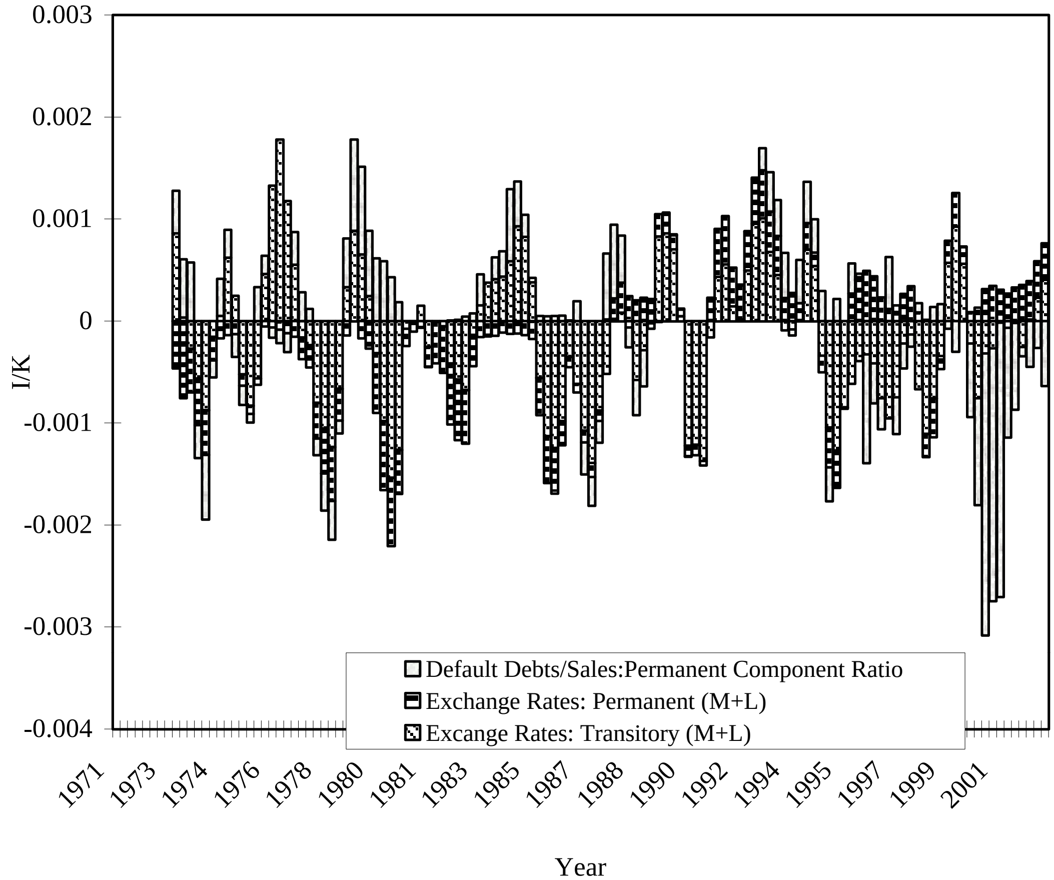


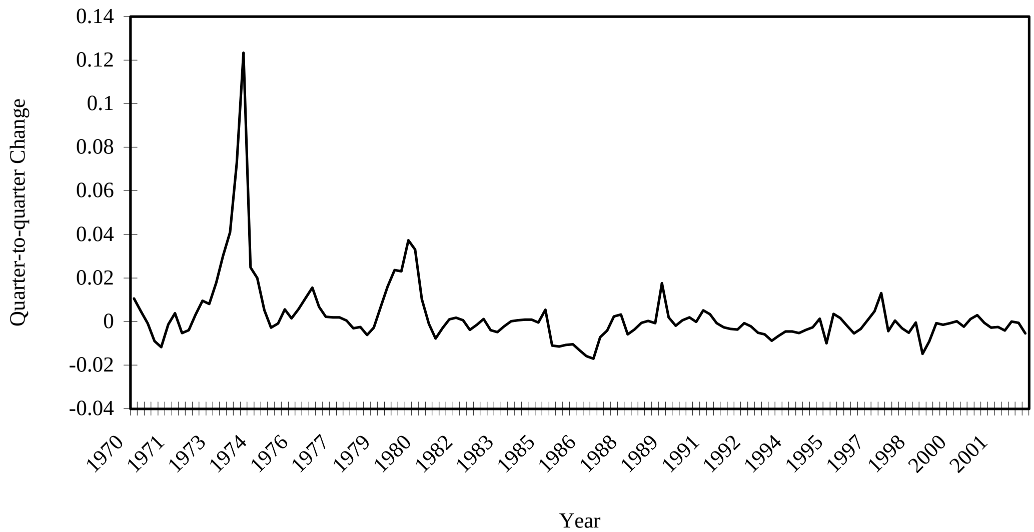
Chart 6. Contributions of Uncertanty Variables to Investment



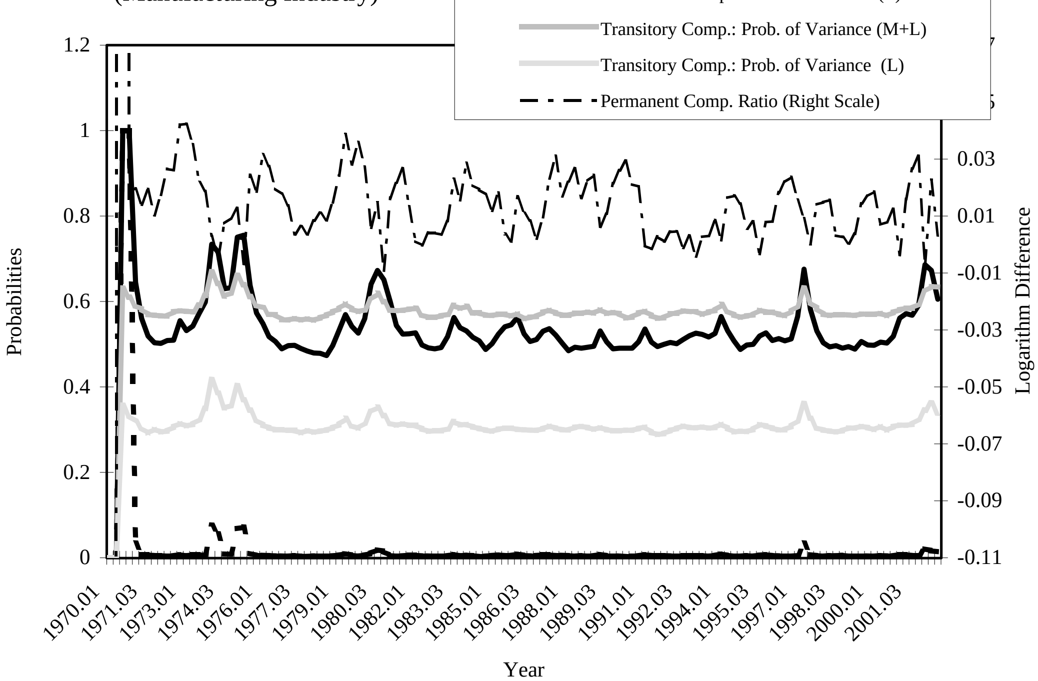
Note 1. Moving average of three quarters.

Note 2. Positive values refer not to positive effects on investment, but smaller negative effects on investment.

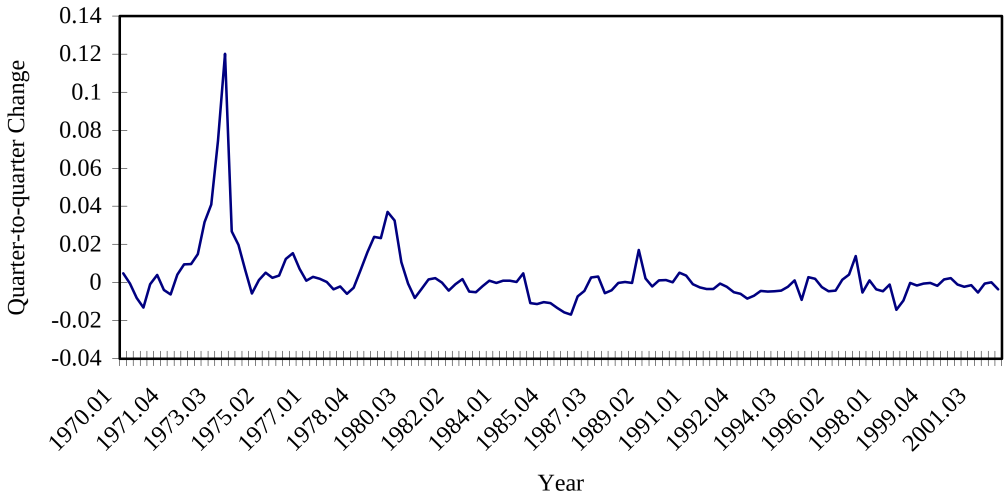
Appendix Chart 1. Real Sales:
(Manufacturing Industry)



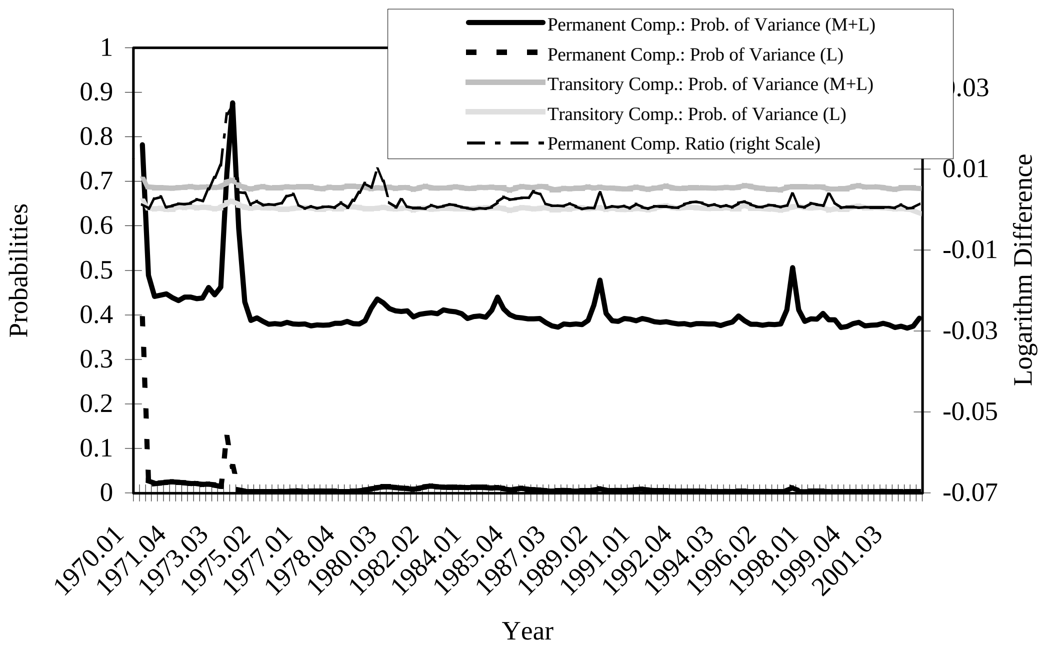
Appendix Chart 2. Real Sales:
(Manufacturing Industry)



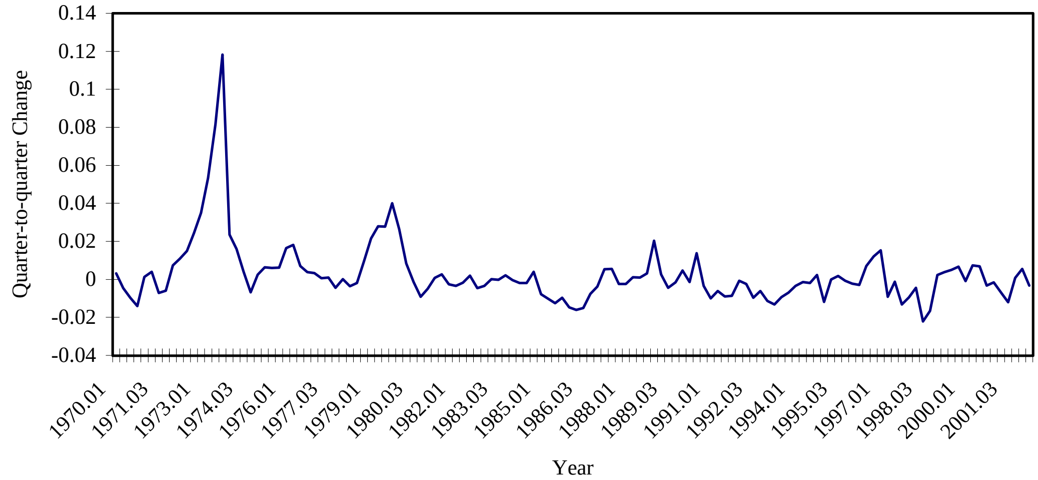
Apendix Chart 3. Output Price:
(Manufacturing Industry)



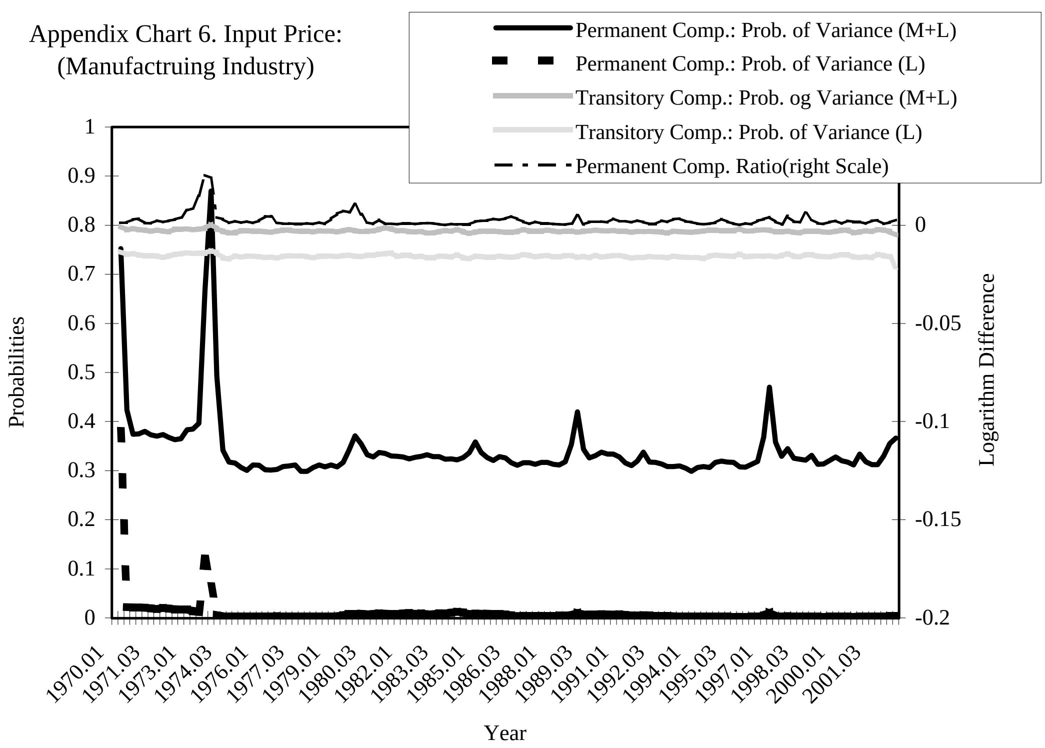
Appendix Chart 4. Output Price:
(Manufacturing Industry)



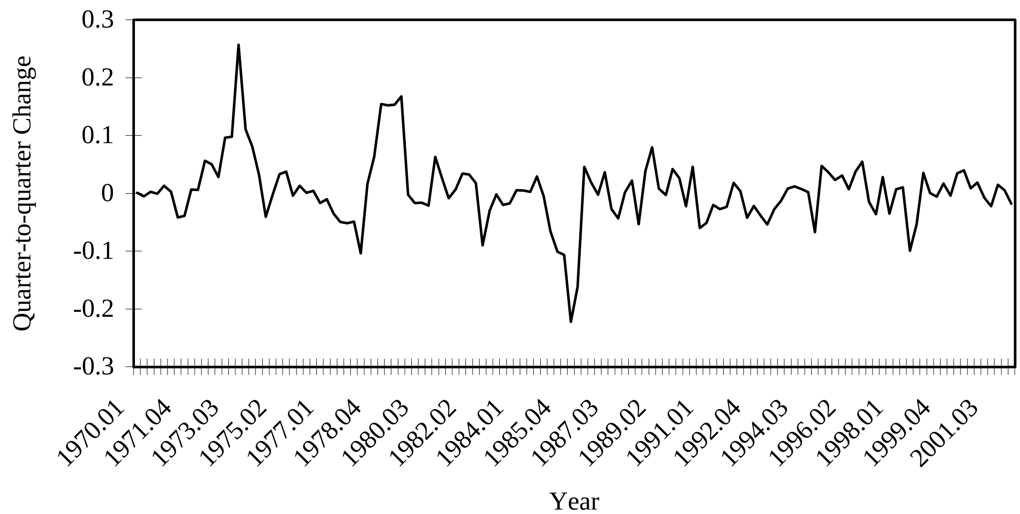
Appendix Chart 5. Input Price:
(Manufacturing Industry)



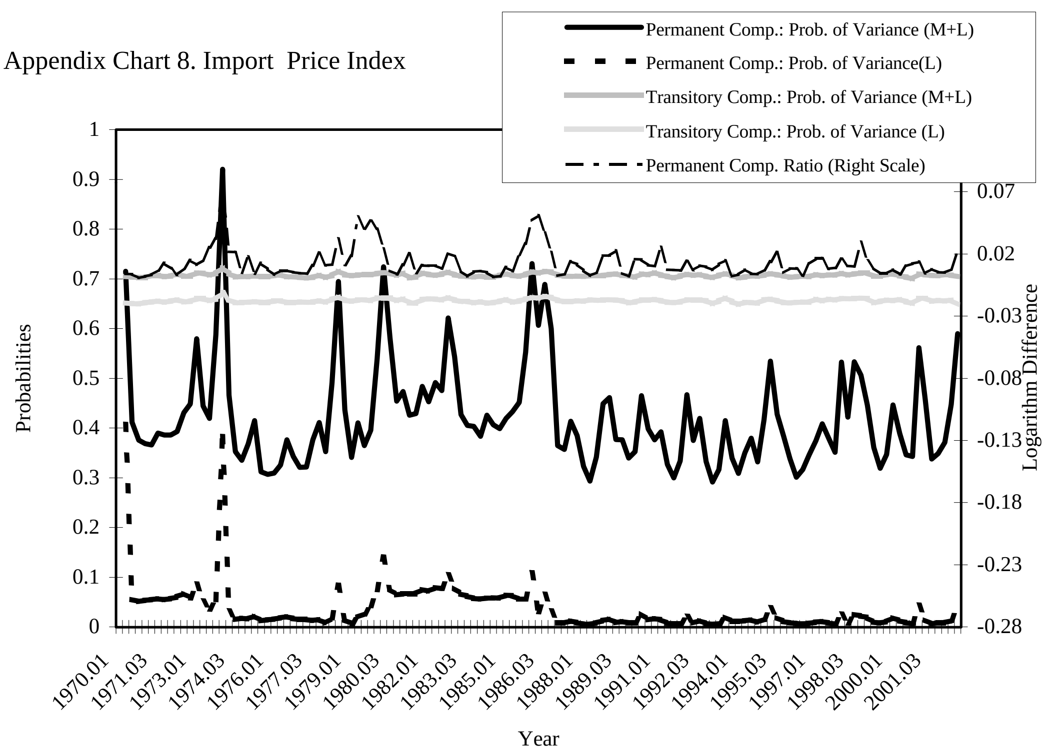
Appendix Chart 6. Input Price:
(Manufactruing Industry)



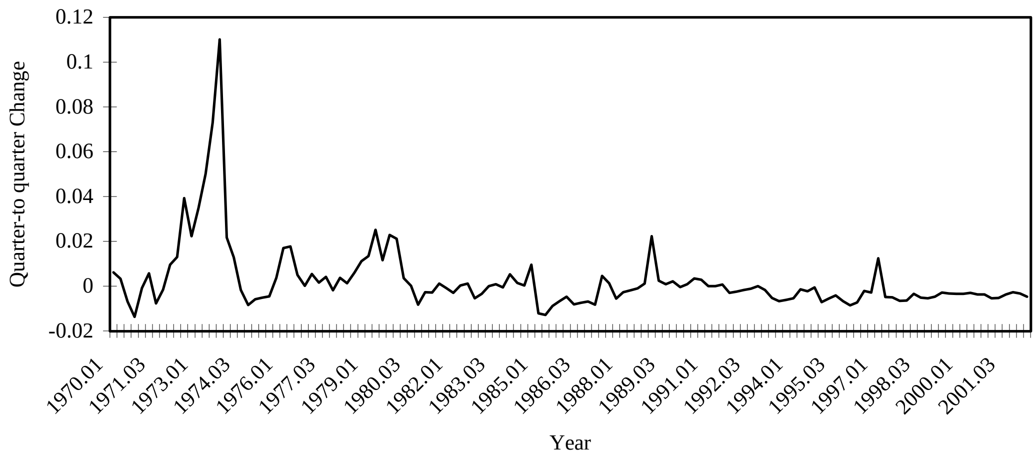
Appendix Chart 7. Import Price Index



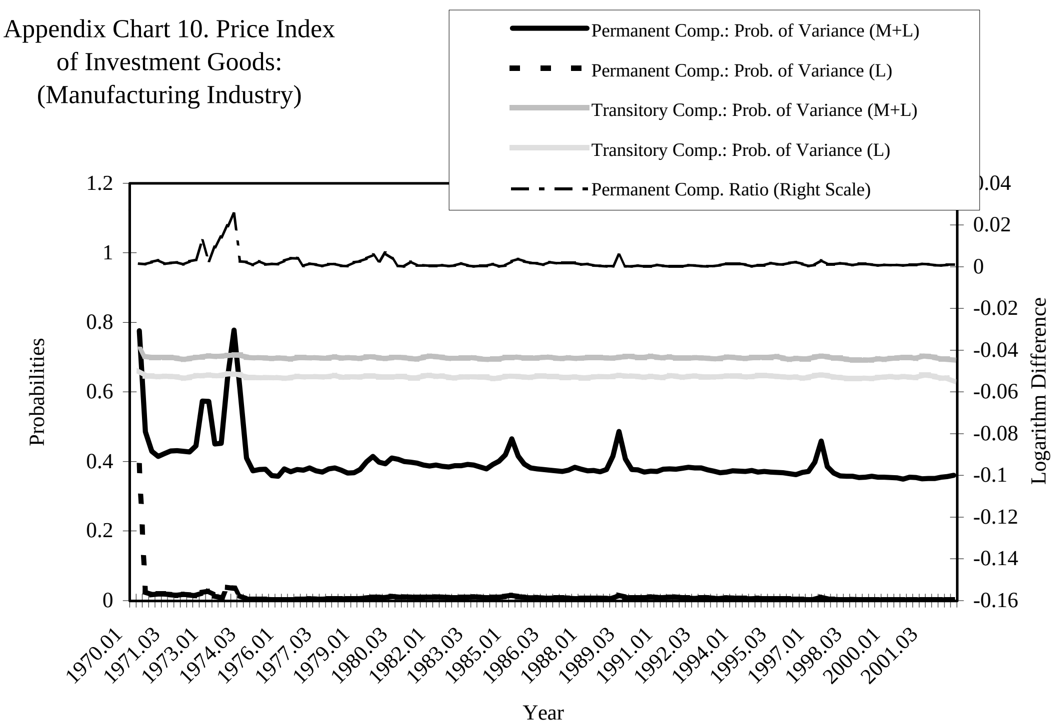
Appendix Chart 8. Import Price Index



Appendix Chart 9. Price Index of Investment Goods::
(Manufacturing Industry)

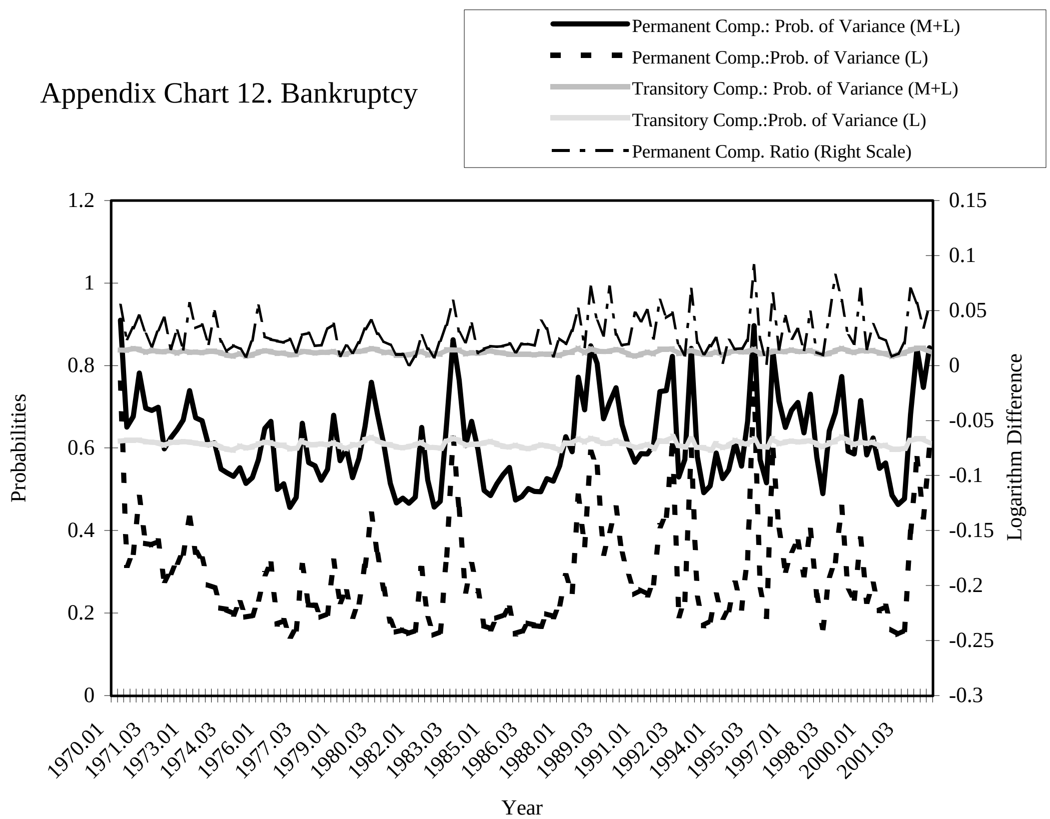


Appendix Chart 10. Price Index
of Investment Goods:
(Manufacturing Industry)

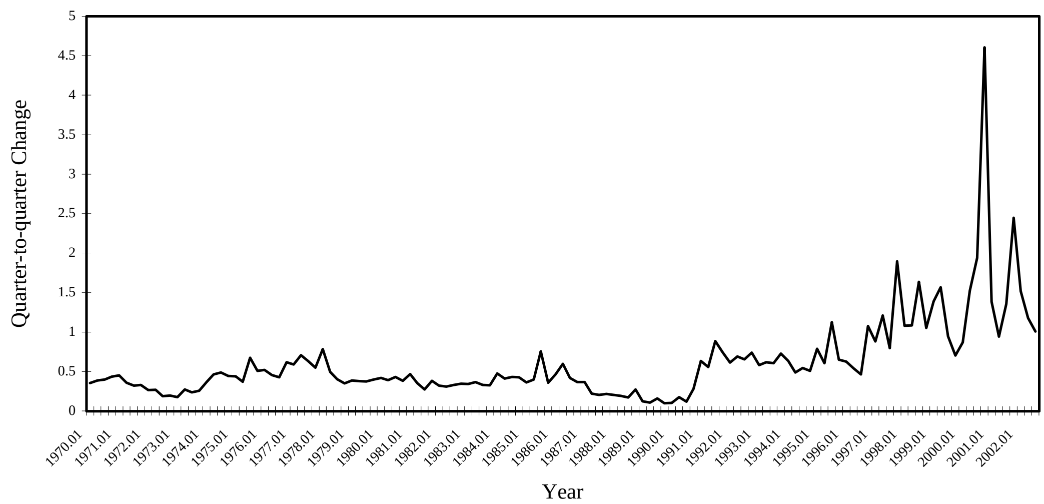


Appendix Chart 11. Bankruptcy

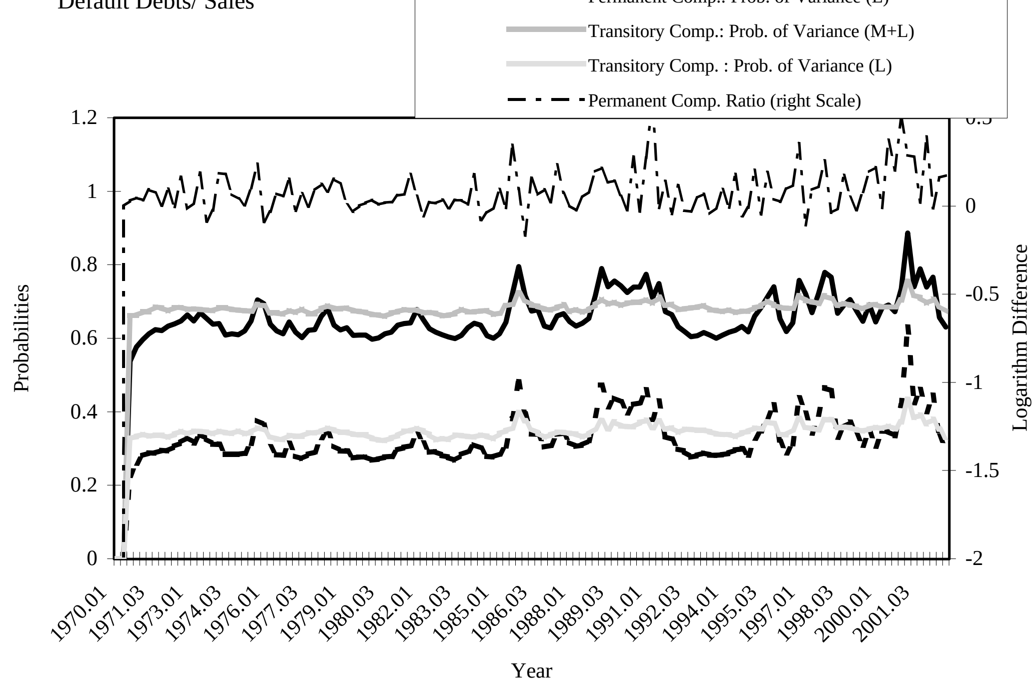
Appendix Chart 12. Bankruptcy



Appendix Chart 13. Default Debts/ Sales

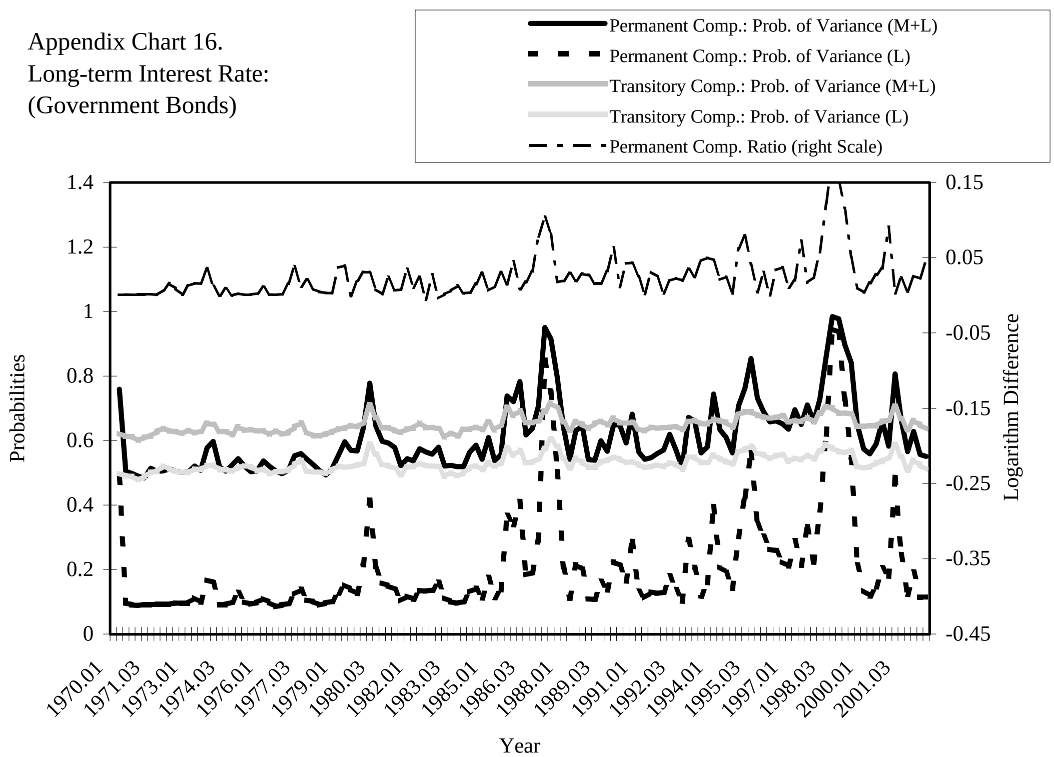


Appendix Chart 14.
Default Debts/ Sales

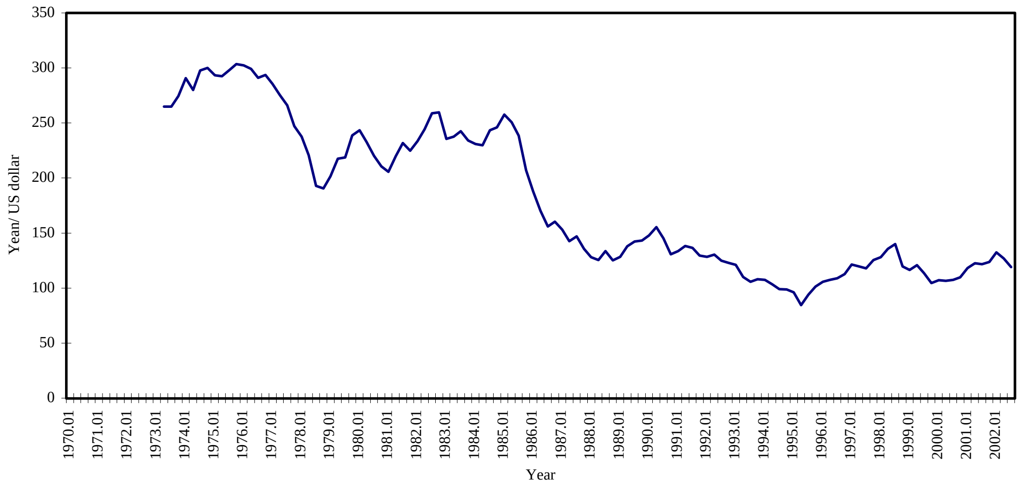


Appendix Chart 15. Long-term Interest Rate:
(Government Bonds)

Appendix Chart 16.
Long-term Interest Rate:
(Government Bonds)



Appendix Chart 17. Exchange Rate (Yen/ US dollar)



Appendix Chart 18.
Exchange Rate (Yen/ US Dollar)